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OBTAINING AND STUDYING CHALCOGENIDE SEMICONDUCTOR FILMS FOR MAKING SOLAR PHOTOCELLS

The possibiliti of obtaining CdS films by chemical and electrochemical methods, as well as CdTe films by magnetron method for manufacturing solar cells with heterostructure CdS / CdTe. The transparent glass was used as a conductive substrate coated with a layer of In_2O_3:Sn (ITO).

Keywords: chalcogenide semiconductors, solar cells, galvanostatic method, magnetron method.

UDC 620.172.24

A.ZH. KHACHATRIAN, A.I. SOGHOMONYAN, A.YU. ALEKSANYAN

THE HARTMAN EFFECT AND THE PROCESS OF A BOUND STATE FORMATION

In this work, the evolution of wave packets scattering on a field of a one-dimensional potential is considered. The scattering potential is taken as a system of two similar rectangular barriers and the wave packets are constructed on the basis of the scattering wave functions. The process of a quasi-bound state appearance in the region between the barriers is investigated. We investigate the time characteristics, such as the bound state appearance time and its life or the delay time of this process. In particular, the dependence of the time characteristics on the width of rectangular barriers of the scattering potential is considered. It is shown that when the carrier energy of wave packets coincides with the energy value of resonance transmission when in the volume of the scattering potential a quasi-bound state system is formed.

Keywords: scattering problem, bound state formation, appearance and delay times.

Introduction. In this work, we consider the time evolution of a wave process initially having the form of two wave packets falling from the left and right sides on a one-dimension scattering potential. Under certain conditions, when the carrier energy of the wave packets equals to the energy of the resonance transmission, the wave process brings to the formation of a quasi-bound state into the value of the scattering potential. In particular, we consider the genesis and collapse process of a bound state into the value of a simple rectangular well which is located inside the scattering potential, namely in the region between two identical rectangular potentials. Such a scattering system, when the width of the rectangular potentials takes an infinitely large value, transforms to a simple potential well. We investigate the time characteristics of the formation and delay processes of a quasi-bound state as functions of the width of the scattering system barriers. In particular, in the limit of infinite ly wide barriers, when the scattering potential having from two identical rectangular barriers takes a quantum well form, the process time evolution is considered.

In outlines of the time evolution of the above described process seems to be clear. Some part of the falling wave perturbation reflects from the scattering system. The remaining part enters into the region of the barriers where a quasi-bound state can appear. As the time characteristics of the discussed wave process, we consider the entry time of the wave packets into the scattering potential volume and the life time or decay time of the appeared quasi-bound state. The entry time is defined as the difference between the achievement times of the maximum value of the wave perturbation in the well center, which are calculated for two cases of scattering potential present and absent. The delay process of a quasi-bound is directly started after its appearance process, i.e. the final time moment of the appearance time is the start time moment from which the life time is calculated. The final stage of the delay process is considered for the time moment when the diverging wave perturbation immediately at the borders of the scattering potential takes the maximum value.

Theory. It is well known that a quantum particle motion in a potential field $U(x)$ is described by means of the time-dependent Schrodinger equation, which for the case of a one-dimensional motion has the form:

$$i\hbar\frac{\partial}{\partial t}\Phi(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+U(x)\right]\Phi(x,t), \tag{1}$$

where $\Phi(x,t)$ is the wave function. This equation has to be collaterally considered with an initial condition, defining the space dependence of the wave function for an initial time moment:

$$\Phi(x,0)=\Phi_0(x), \tag{2}$$

where $\Phi_0(x)$ is given. The standard requirement imposed on a wave function is the normalization condition:

$$\int_{-\infty}^{+\infty}\Phi(x,t)\Phi^*(x,t)\,dx=1. \tag{3}$$

As a function of time, the dependence of function $\Phi\left(x,t\right)$ is usually divided into two types. First of them is the harmonic form of the time dependence and the second one is all the others. For time harmonic solitons, the wave function is considered as

$$\Phi\left(x,t\right)=\exp\left\{-iE\left(k\right)t/\hbar\right\}\Psi(x,k),$$

where $\Psi(x,k)$ satisfies the stationary Schrodinger equation:

$$\frac{d^2\Psi(x)}{dx^2}+\left(k^2-u(x)\right)\Psi(x,k)=0 \tag{4}$$

and $k=\sqrt{2mE(k)}/\hbar$, $u(x)=2mU(x)/\hbar^2$. The function $u(x)$, which is usually called a potential, can take both positive and negative values. Further, the potentials vanishing at infinites will be considered only:

$$u(x\rightarrow\pm\infty)=0.$$

The above given equation can have two solutions. The first corresponds to an infinite motion, when the wave function is normalized to delta-function and the second one describes the finite motion or bound states when the wave function is normalized to a finite quantity. Note that for the last case the energy can take discrete values only, and for the existence of bound states in the same places, the potential has to have negative values.

The wave functions of infinite and finite motions have different natures, which are seen from the difference between their normalization conditions:

$$\int_{-\infty}^{\infty}\psi(x,k)\psi(x,-k')\,dx=\delta(k-k'),\quad \int_{-\infty}^{\infty}\phi(x,\chi_n)\phi(x,\chi_m)\,dx=\delta_{nm}.$$

Here we denoted the wave functions of an infinite motion as ψ and for a bound state as ϕ. Note that for the bound states in Eq. (4), quantity k^2 takes a negative value, so that $k^2=-\chi^2$ and $k=i\chi$, where χ is a real quantity.

As seen from the above written conditions, the wave function of the infinite motion has no dimension, i.e. it is a dimensionless quantity (note that the delta function has a dimension inverse to its argument dimension, for example, $\left[k\right]=1/meter$ and $\left[\delta(k)\right]=meter$). In contrast to that, the wave function of a bound state has a dimension equaling to $1/\sqrt{meter}$.

For the one-dimensional scattering theory, the more interesting solutions of Eq. (4) are the so-called scattering wave functions, corresponding to the left and right scattering problems, which have the following forms of an asymptotic behavior:

$$\psi_l\left(x,k\right)=\frac{1}{\sqrt{2\pi}}\begin{cases}\exp\left\{ikx\right\}+R\left(k\right)\exp\left\{-ikx\right\}, & x\to-\infty,\\ T\left(k\right)\exp\left\{ikx\right\}, & x\to+\infty,\end{cases} \tag{5}$$

$$\psi_r\left(x,k\right)=\frac{1}{\sqrt{2\pi}}\begin{cases}S\left(k\right)\exp\left\{-ikx\right\}, & x\to-\infty,\\ \exp\left\{-ikx\right\}+P\left(k\right)\exp\left\{ikx\right\}, & x\to+\infty\end{cases} \tag{6}$$

where $k>0$ and $T(k)$, $R(k)$ and $S\left(k\right)$, $P\left(k\right)$ are the transmission and reflection amplitudes of the left and right scattering problems, correspondingly . Note that in Eq. (5) and Eq. (6), the factor $1/\sqrt{2\pi}$ provides the normalization condition on the delta function of the scattering wave functions (see, for example, [1]):

$$\int\limits_{-\infty}^{\infty}\psi_l(x,k)\psi_l(x,-k')\,dx=\delta(k-k'),\ \int\limits_{-\infty}^{\infty}\psi_r(x,k)\psi_r(x,-k')\,dx=\delta(k-k'). \tag{7}$$

It is important to mention as well that the left and right scattering functions are orthogonal to each other:

$$\int\limits_{-\infty}^{+\infty}\psi_l\left(x,k\right)\psi_r\left(x,-k'\right)dx=0. \tag{8}$$

The functions $\psi_l(x,k)$ and $\psi_r(x,k)$ are independent solutions of Eq. (4), so its arbitrary solution can be presented by them in a linear combination form. For the transmission and reflection amplitudes of the left and right scattering problems, the following relations take place (see, for example, [2-4]):

$$1-R(k)R(-k)=T(k)T(-k), \tag{9}$$

$$1-P(k)P(-k)=S(k)S(-k), \tag{10}$$

$$P(k)T(-k)+R(-k)S(k)=0, \tag{11}$$

$$S(k)=T(k). \tag{12}$$

Note that for the case, when in Eq. (4), the potential $u(x)$ is a real function, the action sign change of parameter k is equivalent to the complex conjugation action (for example, $\psi_l(x,-k)=\psi_l^*(x,k)$, $R(-k)=R^*(k)$ and so on). Below, we will consider the real potentials only.

As it follows from relations (11), (12), in this case the reflection amplitudes of the left and right scattering problems differ from each other by a phase factor:

$$R(k)=-P^*(k)T(k)/T^*(k). \tag{13}$$

Presenting $T(k)=|T(k)|\exp\{\mathrm{i}\varphi_T(k)\}$ where $|T(k)|$ and $\varphi_T(k)$ are the module and phase of transmission amplitudes, it is easy to see that the last equation takes the form of:

$$R(k)=-P^*(k)\exp\{\mathrm{i}\,2\varphi_T(k)\}. \tag{14}$$

From this equation it particularly follows that

$$|R(k)|=|P(k)| \text{ and } \varphi_R(k)+\varphi_p(k)=\pi+2\varphi_T(k), \tag{15}$$

where $|R(k)|,|P(k)|$ and $\varphi_R(k)$, $\varphi_p(k)$ are the modules and phases of the reflection amplitudes of the left and right scattering amplitudes:

$$R(k)=|R(k)|\exp\{\mathrm{i}\varphi_R(k)\},\ P(k)=|P(k)|\exp\{\mathrm{i}\varphi_P(k)\}.$$

Below we consider the evolution problem of wave packets constructed on the basis of scattering wave functions (5), (6):

$$\Phi(x,t)=\int_0^\infty\left[v_l(k)\psi_l(x,k)+v_r(k)\psi_r(x,k)\right]\exp\{-iE(k)t/\hbar\}dk, \tag{16}$$

where $E(k)=\hbar^2k^2/2m$ and $v_l(k)$, $v_r(k)$ are the coefficients of the expansion spectrum of the wave process $\Phi(x,t)$ conducted on the basis of the scattering functions $\psi_l(x,k)$, $\psi_r(x,k)$. Note that in case of choosing any other basis of orthogonal functions, for example, Fourier waves:

$$\frac{1}{\sqrt{2\pi}}\exp\{i(kx-E(k)t/\hbar)\},\ \frac{1}{\sqrt{2\pi}}\exp\{-i(kx+E(k)t/\hbar)\},$$

the expansion coefficients will depend on t, unless of course, $u(x)=0$ everywhere.

$$v_l(k)=\int_{-\infty}^{+\infty}\Phi_0(x)\psi_l^*(x,k)dx,\ v_r(k)=\int_{-\infty}^{+\infty}\Phi_0(x)\psi_r^*(x,k)dx. \tag{17}$$

It is easy to check when for the function $\Phi(x,t)$ satisfying condition (3), the spectral coefficients would be chosen so that

$$\int\limits_0^\infty [v_l(k)v_l^*(k)+v_r(k)v_r^*(k)]dk=1 \ . \tag{18}$$

In accordance with (16)-(18), the evolution of the wave perturbation $\Phi(x,t)$ is defined by the form of the functions $v_l(k)$, $v_r(k)$.

Resonance tunneling. As a scattering potential, we will consider the system of two identical rectangular potentials having width d and divided from each other by distance L. It is easy to see that that this scattering potential transforms to a simple rectangular well when $d \to \infty$ (see Fig. 1).

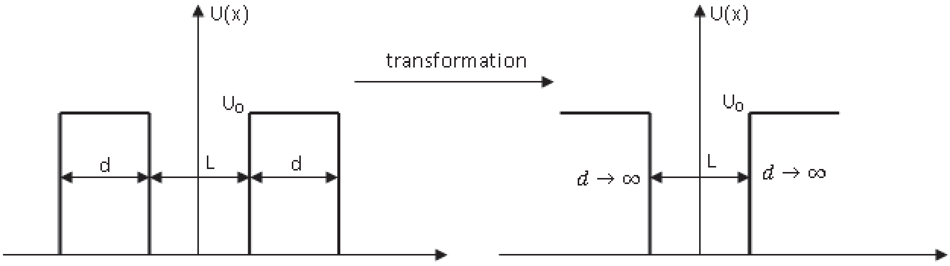

Fig. 1. The set of two barriers when $d \to \infty$ *transforms to a simple well*

The scattering amplitudes of any one-dimensional potential presented by means of two parts can be written by means of the scattering amplitudes corresponding to these parts (see, for example, [5]). So, the system transmission amplitude has the form:

$$T=\frac{t_I t_{II}}{1-p_I r_{II}} \tag{19}$$

where the indexes *I* and *II* correspond to the left and right barriers of the scattering potential. So, p_I is the reflection amplitude of the first barrier determined from the right scattering problem and r_{II} is the reflection amplitude of the second barrier determined for the left scattering problem. Applying the general relation (13) to the first barrier ($p_I=-r_I^* t_I/t_I^*$), and taking into account that the barriers of the scattering potential are identical ($t_I=t_{II}=t$ and $r_I=r_{II}\exp\{-i2k(L+d)\}=r$), for (19), one can write:

$$T = \frac{|t|^2 \exp[2i\varphi_t]}{1 + |r|^2 \exp\left[2i\varphi_{res}\right]}, \tag{20}$$

where φ_t is the phase of the transmission amplitude of a single barrier ($t = |t|\exp[i\varphi_t]$), and we can make a notation:

$$\varphi_{res} = kL + kd + \varphi_t. \tag{21}$$

It is easy to see that when

$$\varphi_{res} = \pi / 2, \tag{22}$$

the exponent in the dominator of (20) equals to -1. Taking into account the fact that for any value of k, the equality $1 - |r|^2 = |t|^2$ takes place, one can check that Eq. (22) defines the resonance values of k_n, i.e. when $T(k_n) = 1$. So, Eq. (22) is the transmission resonance condition for a two-barrier scattering potential.

In the region of the sun-barrier scattering ($E(k) < U_0$), the modules and phases of the transmission and reflection amplitudes of a rectangular barrier have the forms:

$$\frac{1}{|t|^2} = \cosh^2\{\chi d\} + \left(\frac{k^2 - \chi^2}{2\chi k}\right)^2 \sinh^2\{\chi d\} \ , \tag{23}$$

$$\frac{|r|^2}{|t|^2} = \left(\frac{k^2 + \chi^2}{2\chi k}\right)^2 \sinh^2\{\chi d\}, \tag{24}$$

$$\varphi_t(k) = -kd + arctg\left[\frac{k^2 - \chi^2}{2\chi k} \operatorname{tgh}\{\chi d\}\right] \tag{25}$$

where $\chi = \sqrt{2m(U_0 - E(k))} / \hbar$.

By using Eq. (25), the resonance condition (22), one can write (see, for example, [6, 7]):

$$ctg\{k_n L\} = \frac{k_n^2 - \chi_n^2}{2\chi_n k_n} \operatorname{tgh}\{\chi_n d\}. \tag{26}$$

Here index n mentions that this equality can take place for certain values of k_n only and

$$\chi_n = \sqrt{2mU_0 / \hbar^2 - k_n^2} \ .$$

As it follows from the above mentioned, the resonance values of the wave number depend on the distance between the barriers, the width and the magnitude of the potential of the barriers;

$$k_n = k_n(L, d, U_0), \tag{27}$$

where, as it was mentioned, k_n are the magnitudes of the quasi wave number, corresponding to the resonance tunneling through a scattering potential, when the particle transmits the potential with probability equal to unity.

It should be mentioned as well that when $\chi d \to \infty$, condition (26) transforms to the equation determining the energy spectrum of the simple rectangular well [8]:

$$ctg\{kL\} = \frac{k^2 - \chi^2}{2\chi k}. \tag{28}$$

Since the transmission amplitudes of the left and right scattering problems equal each other (see Eq. (12)) and the reflection amplitudes differ by the phase factor (see Eq. (14)) for the same value of k_n, the resonance tunneling can take place in two directions. If for a given value of k_n a particle falling from the left on a potential resonantly transmits, when for this value k_n, it will resonantly transmit potential falling from the right as well:

$$|T(k_n)| = |S(k_n)| = 1 \text{ and } |R(k_n)| = |P(k_n)| = 0. \tag{29}$$

Here we consider the wave packets with spectral composition of k near to the magnitude of k_n, i.e. taking magnitudes into the interval:

$$k_n - \Delta k \le k \le k_n + \Delta k \tag{30}$$

where $\Delta k << k_n$. Expanding the modules and phases of scattering amplitudes $T(k)$, $R(k)$ and $P(k)$ on series k near the resonance values, one can write:

$$T(k) = \exp\{\mathrm{i}\left(\varphi_T(k_n) + \varphi_T'(k_n)(k - k_n)\right)\}, \ R(k) = P(k) = 0, \tag{31}$$

$$k^2 = (k_n + (k - k_n))^2 \approx k_n^2 + 2k_n(k - k_n)\,. \tag{32}$$

Note that near the resonance modules of the transmission and reflection, amplitudes take their maximum and minimum values (see Eq. (29)), so that near the resonance

$$\partial\left|T(k_n)\right|\partial k = \partial\left|R(k_n)\right|\partial k = \partial\left|P(k_n)\right|\partial k = 0\,.$$

In accordance with equations (31), (32), in the region of k near to k_n (30), the scattering wave functions can be presented:

$$\psi_l\left(x,k\right) = \frac{1}{\sqrt{2\pi}}\begin{cases} \exp\left\{ikx\right\}, \quad x \to -\infty, \\ \exp i\left\{kx + \varphi_T(k_n) + \varphi_T'(k_n)(k - k_n)\right\}, \quad x \to +\infty, \end{cases} \tag{33}$$

$$\psi_r\left(x,k\right) = \frac{1}{\sqrt{2\pi}}\begin{cases} \exp i\left\{\varphi_T(k_n) + \varphi_T'(k_n)(k - k_n) - kx\right\}, \quad x \to -\infty, \\ \exp\left\{-ikx\right\} \qquad x \to +\infty. \end{cases} \tag{34}$$

Below we consider the wave packets, having the following spectral composition:

$$v_l(k) = \frac{\mathrm{e}^{ik(L+d)}}{4\sqrt{\Delta k}}\begin{cases} 0, \quad k < k_n - \Delta k, \\ 1, \; k_n - \Delta k < k < k_n + \Delta k, \\ 0, \quad k > k_n + \Delta k, \end{cases} \tag{35}$$

$$v_r(k) = \frac{\mathrm{e}^{-ik(L+d)}}{4\sqrt{\Delta k}}\begin{cases} 0, \quad k < k_n - \Delta k, \\ 1, \; k_n - \Delta k < k < k_n + \Delta k, \\ 0, \quad k > k_n + \Delta k. \end{cases} \tag{36}$$

It is easy to check that the chosen forms of the spectral coefficients $v_l(k)$, $v_r(k)$ satisfy condition (18) and prove the certain form of initial perturbation $\Phi_0\left(x\right)$ (see Eq. (2)). Namely, at the initial time moment $t = 0$, there are two wave packets with maximums of the border points of the scattering potential $x = -L - d$ and $x = L + d$, i.e. in the left point of the first barrier and in the right point of the second one.

In accordance with the above given statement of the wave evolution problem we have done some calculations relating to the time characteristics of a quasi-bound state appearance τ^r and its decay τ^p. In the Figure, we present the time as a dimensionless quantity in the units of u_g / a. The parameters of the quantum well was chosen as $2mU_0a^2 / \hbar^2 = 7$, which has three bound states. It is easy to see that when the width of the barriers tends to infinite, the appearance time tends a finite value, while the decay time limits to infinite.

Conclusion. As it follows from the obtained result, any bound state formed into the potential volume is a standing wave packet which arises due to the certain wave scattering process.

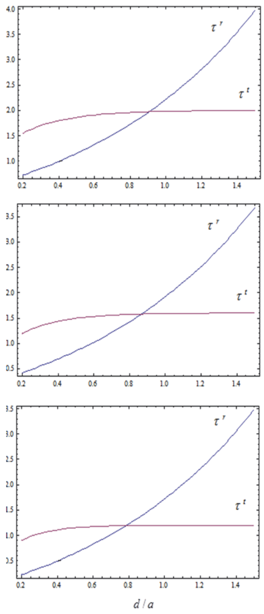

Fig. 2. The time characteristics of the considered wave process

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ՀԱՐՏՄԱՆԻ ԷՖԵԿՏԸ ԵՎ ԿԱՊՎԱԾ ՎԻՃԱԿԻ ԱՌԱՋԱՑՈՒՄԸ

Դիտարկվել է միաչափ պոտենցիալի դաշտի վրա ցրվող ալիքային ծրարների էվոլյուցիայի խնդիրը: Ցրող պոտենցիալը դիտարկվել է երկու նույնանման ուղղանկյուն պոտենցիալներից կազմված համակարգի տեսքով և ալիքային ծրարներից կազմված ցրման ալիքային ֆունկցիաների միջոցով: Հետազոտվում է միջարգելքային տիրույթում քվազիկապված վիճակի առաջացման պրոցեսը: Հետազոտվում են տվյալ պրոցեսի ժամանակային բնութագրերը. քվազիկապված վիճակի առաջացման ժամանակը և նրա կյանքի կամ տարրալուծման ժամանակը: Մասնավորապես, դիտարկվել է ժամանակային բնութագրերի կախումը ցրող դաշտի ուղղանկյուն արգելքների հաստությունից: Ցույց է տրվել, որ երբ ալիքային ծրարների կրող էներգիան լինում է հավասար ռեզոնանսային անցման էներգիային, ապա ցրող պոտենցիալի ծավալում ձևավորվում է կապված վիճակ:

Առանցքային բառեր. ցրման խնդիր, կապված վիճակի առաջացում, առաջացման և տարրալուծման ժամանակ:

А.Ж. ХАЧАТРЯН, А.И. СОГОМОНЯН, А.Ю. АЛЕКСАНЯН

ЭФФЕКТ ХАРТМАНА И ПРОЦЕСС ОБРАЗОВАНИЯ СВЯЗАННОГО СОСТОЯНИЯ

Рассматривается эволюция волновых пакетов рассеивающегося в поле одномерного потенциала. Рассеивающий потенциал рассмотрен в виде системы из двух идентичных прямоугольных барьеров, а волновые пакеты сконструированы на базе волновых функций рассеяния. Исследуется процесс образования квазисвязанного состояния в межбарьерной области. Изучены временные характеристики данного процесса, такие как время появления связанного состояния и время его жизни или распада. В частности, рассматривается зависимость временных характеристик от толщины прямоугольных потенциалов рассеивающего потенциала. Показано, что в случае, когда несущая энергия волнового пакета совпадает с энергией резонансного туннелирования, в объёме рассеивающего потенциала формируется квазисвязанное состояние.

Ключевые слова: задача рассеяния, образование квазисвязанного состояния, времена образования и распада.