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A SYSTEM FOR TRACKING THE SUN FOR THE MIRROR PARABOLOID CONCENTRATORS OF OPTICAL RADIATION

A system for tracking the sun for the mirror paraboloid concentrators of optical radiation is described. It is shown how it is possible, through the tracking system, to use the energy of solar radiation more efficiently. The methods are described, at whose application it is expedient to use paraboloid concentrators.

Keywords: system of tracking the sun, paraboloid mirror, concentrator.

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SIZE QUANTIZATION OF HOLES IN A THIN FILM

We describe a new method for determining the energy eigenstates of a quantum well in the multiple band envelope function approximation. A unitary transformation is used to diagonalize the 4×4 Luttinger-Kohn Hamiltonian into 2×2 blocks, making it more efficient to calculate the quantum well subband structure in both cases of infinite and finite quantum well models. Valence subbands of GaAs/$Ga_{1-x}Al_xAs$ quantum wells are found by solving exactly the multiband effective mass equation for the envelope function, and then we impose boundary conditions for solutions of 2×2 block Hamiltonian. It is shown that the boundary conditions can change the sequence of energy subbands.

Keywords: heavy and light holes, infinite and finite potential barries, energetic spectrum, wave functions.

Introduction

The rapid progress of experimental efforts to fabricate quantum wells, multiple quantum wells and quantum wells supperlattices is a strong impetus to develop theoretical techniques for their study. In this paper. we present a complete account of a new analytical method for determining a simple expression for eigenstates and eigenenergies of the quantum well.

The spectrum of the thin film is well known[4,5]: there are a number of subbands with a parabolic dependence of the energy on the momentum of the electronic motion parallel to the layer plane, the effective mass of the motion being equal to that of the bulk. The energy spectrum of the zincblende semiconductor heterostructures differs considerably from that simple picture[4-9], it shows various anomalies of several types, caused in general by two circumstances. First, there is a strong influence of the hole motion parallel to the heterostructure interface on its

motion normal to the interface and vice versa. Second, in the vicinity of the interface the transformation proceses of different types of holes is possible, e.g. transformation of light holes into heavy holes and so on[4-9].

In any study of the properties of the valence subbands of a quantum well, it is necessary to use a realistic bandstructure model which includes interactions between the bands. For this purpose, all valence subband dispersions are determined using a **kp** method which accounts for mixing between states derived from the HH and LH bands. Band structure calculations for a single quantum well have been performed first for an infinite well[4,5] and latter for finite well corresponding to the band offset within the Luttinger matrix Hamiltonian[12-18].

II. EXACT SOLUTION OF THE EFFECTIVE-MASS EQUATION

The upper valence band states at wave vectors **k** near Γ_8 point are expanded in terms of the four zone center eigenstates composed of the L=1 valence band basis functions |X>, |Y> and |Z> plus spin[1-3]. The HH center states are then described by spin quantum numbers |3/2,±3/2 >, the LH states by |3/2,±1/2 >. The valence band structure of a bulk semiconductor can be described by the following 4×4 Hamiltonian in the envelope function space[3]:

$$H_0(k_x,k_y,\hat{k}_z=-i\frac{d}{dz})=-\frac{\hbar^2}{2m_0}\begin{bmatrix} P+Q & R & -S & 0 \\ R^* & P-Q & 0 & S \\ -S^* & 0 & P-Q & R \\ 0 & S^* & R^* & P+Q \end{bmatrix} \tag{1}$$

with $P=\gamma_1(k_x^2+k_y^2+k_z^2), Q=\gamma_2(k_x^2+k_y^2-2k_z^2), S=2\sqrt{3}\gamma_3(k_x-ik_y)k_z, R=-\sqrt{3}\gamma(k_x-ik_y)^2.$

In Hamiltonian (1) we apply the axial approximation[12,14] which consists in assuming $\gamma_1 > \gamma_2 \approx \gamma_3$.

The exact solution of the effective mass equation (finite quantum well)

We consider an isolated quantum well (QW) grown in a <100> direction, which we take along the quantization axis Z. The well extends from –a/2 to a/2.

In order to simplify, we transform the Hamiltonian (1) by an unitary transformation, to two independent 2×2 dimensional matrices[18], which represent two spin degenerate states of holes, respectively

$$H_0(k_x,k_y,\hat{k}_z=-i\frac{d}{dz})=-\frac{\hbar^2}{2m_0}\begin{bmatrix} P+Q & |R|-i|S| & 0 & 0 \\ |R|+i|S| & P-Q & 0 & 0 \\ 0 & 0 & P-Q & -|R|+i|S| \\ 0 & 0 & -|R|-i|S| & P+Q \end{bmatrix}+U_0(z) \tag{2}$$

where $U_0(z)=U_0\Theta(z-a/2)\Theta(z+a/2)$ is a square well potential which vanishes inside the well and equals U_0 in the barriers. $\bar{\gamma}=(\gamma_2+\gamma_3)/2$ in (2). Thus the problem of

calculating a 8×8 dimensional matrix of the linear equation system is simplified to a problem of calculating two 4×4 dimensional matrices separately . The Hamiltonian (2) acts on a four component envelope function $\Psi = (\psi_1, \psi_2, \psi_3, \psi_4)$, and the electronic wave functions are given by:

$$\begin{aligned} &\Psi(\vec{r}) = \psi_1(\vec{r})\big|1\big\rangle + \psi_2(\vec{r})\big|2\big\rangle + \psi_3(\vec{r})\big|3\big\rangle + \psi_4(\vec{r})\big|4\big\rangle \equiv \\ &e^{i(xk_x + yk_y)}\left[\psi_1(z)\big|1\big\rangle + \psi_2(z)\big|2\big\rangle + \psi_3(z)\big|3\big\rangle + \psi_4(z)\big|4\big\rangle\right] \end{aligned} \tag{3}$$

The potential $U_0(z)$ breaks the translation symmetry along Z, however, k_x and k_y (or $k_\parallel = \sqrt{k_x^2 + k_y^2}$) remain good quantum numbers. The effective mass equation $\hat{H}\Psi = E\Psi$ must be supplemented by boundary conditions at each interface. In the approximation that the Bloch functions (4) are equal in the two materials, boundary conditions can be expressed in terms of the envelope functions alone.

In principle, finding eigenfunctions for the matrix of size 4 × 4 would lead to a problem of their eigenfunctions for the matrix 2 × 2, that is, we have to choose, the upper or the lower block in Hamiltoniane (2). From the structure of the coupled equations according to Hamiltonian (2) there are four linearly independent solutions for both 2×2 blocks, which have the following from:

$$\begin{cases} \psi_1(z) = c_1 \sin k_h z + c_2 \cos k_h z + c_3 \sin k_l z + c_4 \cos k_l z \\ \psi_2(z) = c_1(-m_h^U \sin k_h z - l_h^U \cos k_h z) + c_2(l_h^U \sin k_h z - m_h^U \cos k_h z) + \\ \qquad c_3(-m_l^U \sin k_l z - l_l^U \cos k_l z) + c_4(l_l^U \sin k_l z - m_l^U \cos k_l z) \\ \psi_3(z) = c_5 \sin k_h z + c_6 \cos k_h z + c_7 \sin k_l z + c_8 \cos k_l z \\ \psi_4(z) = c_5(-m_h^L \sin k_h z - l_h^L \cos k_h z) + c_6(l_h^L \sin k_h z - m_h^L \cos k_h z) + \\ \qquad c_7(-m_l^L \sin k_l z - l_l^L \cos k_l z) + c_8(l_l^L \sin k_l z - m_l^L \cos k_l z) \end{cases} \tag{4}$$

where $k_{h;l} \equiv k_{zh;l} = \sqrt{\dfrac{E}{\gamma_1 \mp 2\gamma_2} - k_\parallel^2}$, $m_{h;l}^{U;L} = \dfrac{M}{m_\pm^0 + m_\mp k_{h;l}^2}$; $l_{h;l}^{U;L} = \dfrac{k_{h;l} L}{m_\pm^0 + m_\mp k_{h;l}^2}$; $m_\pm^0 = (\gamma_1 \pm \gamma_2)k_\parallel^2 - E$; $m_\mp = \gamma_1 \mp 2\gamma_2$; index U- corresponds to the upper 2×2 block Hamiltonian and L- to the lower one, $M = \sqrt{3}\overline{\gamma}k_\parallel^2$, $L = 2\sqrt{3}\gamma_3 k_\parallel$, E is written in the unit $\dfrac{\hbar^2}{2m_0}$.

To determine the eigenvalues and eigenfunctions of the Schrödinger equation in the barrier material (outside the well) it is necessary to use the Hamiltonian (2) after replacing the well material parameters $(\gamma_1, \gamma_2, \gamma_3)$ to the relevant material parameters of the barrier, and also must be replaced E by $U_0 - E$.

The exact solution of the effective mass equation (infinite quantum well)

We direct the axis z perpendicular to the film surface. Let the boundary of the film be located at $z = \pm a/2$. The energy spectrum in the case of infinite well model will be determined from the Schrodinger equation $H\Psi = E\Psi$, with boundary conditions of the $\Psi(-a/2) = \Psi(a/2) = 0$. These boundary conditions lead to a system of eight homogeneous equations for the unknown coefficients. The double Cramer degeneration leads to two identical systems of four equations for the coefficients $D^{4\times4}(k, E)$. Expanding this determinant, we obtain the equation for determining the energy spectra of size quantization states, which is exactly the same with the Nedorezov results[4,5] :

$$1 - \cos(k_h a)\cos(k_l a) - \frac{l_h^2 + l_l^2 + (m_l - m_h)^2}{2 l_h l_l}\sin(k_h a)\sin(k_l a) = 0 \qquad (5)$$

The energy spectrum is as follows: $E_n^{h,l}(k_\parallel) = E_n^{h,l}(0) + \frac{\hbar^2 k_\parallel^2}{2m_n^{h,l}}$ where $E_n^{h,l}(0) = \frac{\pi^2\hbar^2 n^2}{2m_{h,l}a^2}$, that means, at $k_\parallel = 0$ there are two independent systems of levels, i.e. heavy and light holes are quantized independently.

Results and Discussions

The results of our valence-band calculations are shown in Figures 1 and 2, referring to a GaAs/$Ga_{1-x}Al_xAs$ quantum wells having a well width of 20, 50, 100 and 150Å and with an aluminum mole fraction x=0.3.

The valence-subband structure is seen to be very complicated and some of the bands are seen to have negative zone center effective masses. This complicated structure is the result of strong interactions between different subbands due solely to mixing of HH and LH states at a nonzero value of $\mathbf{k}_{||}$ by the off-diagonal terms Hνν' of Hamiltonian (2). At points away from the zone center, the increasing strength of the level repulsion interaction between the subband states with increasing $k_{||}$ give a complicated band structure. In particular, the increasing strength of the level repulsion interaction between LH1 and HH2 gives rise to the negative zone center LH1 effective mass (Fig.1). In the absence of coupling by off-diagonal terms in Hνν', the subbands would all be parabolas. At the zone center, the off-diagonal terms in Hνν' vanish and the HH and LH states are uncoupled. Thus, the $\mathbf{k}_{||}$=0 states can be labeled LHn or HHn for the light- or heavy-hole state.

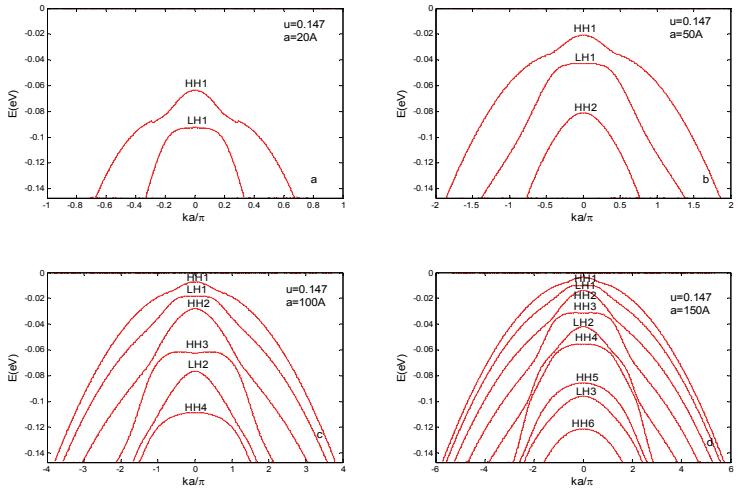

Fig. 1 (finite barrier case: U_0=147.12meV). The valence subband structure for GaAs /$Ga_{0.7}Al_{0.3}As$ quantum well having thicknesses 20, 50, 100 and 150Å and using the axial case of warping: when γ_2 and γ_3 are replaced by the average $\gamma' = (\gamma_2 + \gamma_3)/2$ in Hamiltonian (1) and (2)

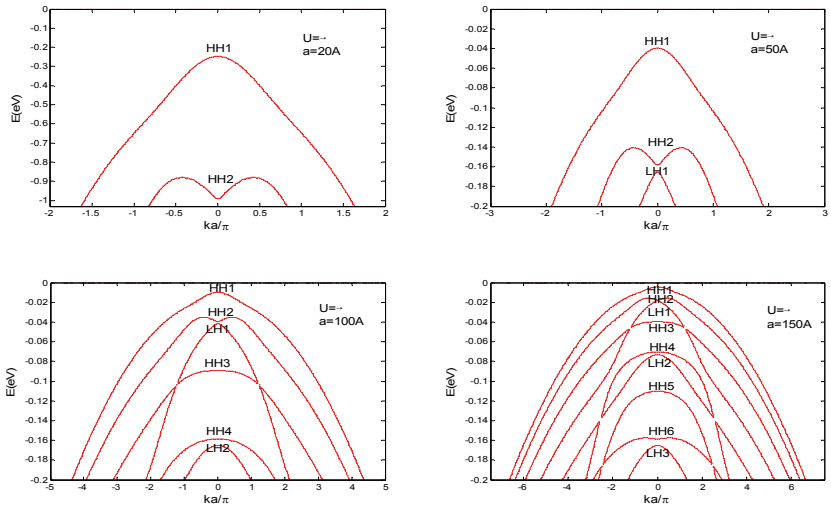

Fig. 2 (infinite barrier case: U_0=∞). The valence subband structure for GaAs /$Ga_{0.7}Al_{0.3}As$ quantum well having thicknesses 20, 50, 100 and 150Å and using the axial case of warping: when γ_2 and γ_3 replaced by the average $\gamma' = (\gamma_2 + \gamma_3)/2$ in Hamiltonians (1) and (2)

In all these valence subband energy dispersions similar non parabolicities of the valence subbands is observed, but the most distinguishing features is the different order of these bands. As can be seen from Figures 1 and 2, the sequence of the valence subbands can seriously change with the size of the quantum well. In

the case of a quantum well size a= 100Å, we have the following location of valence subbands: 1h, 2h, 1l, 3h ,. . ., This location is changed as follows: 1h, 1l, 2h, 3h ,. . . with the axial approach.

In the formation of the valence subbands in the quantum well the quantum model of the well is also important, i.e. the spatial limitation of quasiparticles occurs in different ways between the walls of the well with infinite or finite height and yield different quantitative results. When a = 20A, two energy subbands in a quantum well of infinite depth are formed on the energy scale is about seven times higher than the barrier in a model with finite barrier height. Increasing the size of these wells, the energy scales are close to each other in different models.

Conclusions

We have studied hole subbands of GaAs/$Ga_{1-x}Al_xAs$ quantum wells. Within the effective mass approximation, the energy levels and envelope functions can be determined exactly by solving the effective mass equation in each bulk material. The use of unitary transformation of 4×4 Luttinger Hamiltonian allows to decouple this Hamiltonian into two 2×2 block diagonalized Hamiltonians and to obtain the energy eigenvalues as the zeros of an 8×8 determinant.

Calculations have been done for axial and spheric cases of Luttinger Hamiltonians and it has been found that the sequence of valence subbands is sensitive to the axial or spherical cases of this Hamiltonian. Also, the valence subbands can essentially change their form and zone center effective masses dependent on the infinite or finite model of the quantum well.

This method is also suitable for investigating the phenomena of passing an electric current through different layered nanostructures for superlattice, and also in multiple layers for transistors.

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ԽՈՌՈՉՆԵՐԻ ՉԱՓԱՅԻՆ ՔՎԱՆՏԱՑՈՒՄԸ ԲԱՐԱԿ ԹԱՂԱՆԹՈՒՄ

Ուսումնասիրվել են խոռոչների չափային քվանտացման վիճակները բարակ կիսահաղորդչային թաղանթում: 4×4 չափսի Լատինջերի համիլտոնիանը ունիտար ձևափոխության միջոցով բերվել է 2×2 չափսի բլոկ անկյունագծային տեսքի, որը էապես հեշտացնում է սեփական ֆունկցիաների և սեփական էներգիաների որոշումը: Շրեդինգերի հավասարման լուծումները գրված են անվերջ և վերջավոր պոտենցիալ հորերի մոդելների դեպքում: Հաշվարկները ցույց են տալիս, որ անցումը անվերջ հորի մոդելից վերջավոր հորի դեպքին խոռոչների էներգետիկ սպեկտրը էապես փոխվում է ինչպես քանակապես այնպես էլ որակական ձևով:

Առանցքային բառեր. ծանր և թեթև խոռոչներ, անվերջ և վերջավոր պոտենցիալ հորեր, էներգետիկ սպեկտր, ալիքային ֆունկցիաներ:

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РАЗМЕРНОЕ КВАНТОВАНИЕ ДЫРОК В ТОНКОЙ ПЛЕНКЕ

Исследовано состояние размерного квантования дырок в тонкой полупроводниковой пленке. С помощью унитарного преобразования гамильтониана Латиджера с размерностью 4×4 приведен к блоку диагонального вида с размерностью 2×2, что значительно облегчает определения собственных функций и собственных энергий. Решения уравнения Шриденгера написаны для конечной и бесконечной моделей потенциальной ямы. Расчеты показали, что при переходе от модели бесконечной ямы к случаю конечной ямы энергетический спектр дырок значительно изменяется как качественно, так и количественно.

Ключевые слова: тяжелые и легкие дырки, бесконечный и конечный потенциальные барьеры, энергетический спектр, волновые функции.