Վ.Ա. ՄԻՐԶՈՅԱՆ, Գ.Ս. ՄԱՉԿԱԼՅԱՆ, Ա.Ռ. ՆԱԶԱՐՅԱՆ

ԵՐԵՔ ԿՈՉԱՓԱՆԻ ԵՎ ԿՈՐՈՒԹՅԱՆ ԳԼԽԱՎՈՐ ՎԵԿՏՈՐՆԵՐԻ ԵՐԿՈՒ ԽՈՒՄԲ ՈՒՆԵՑՈՂ ՐԻՉՉԻ - ԿԻՍԱՍԻՄԵՏՐԻԿ ԵՆԹԱԲԱԶՄԱՁԵՎՈՒԹՅՈՒՆՆԵՐԻ ՄԱՍԻՆ

Էվկլիդեսյան տարածություններում հետազոտվում է երեք կոչափանի նորմալ հարթ ռիչչի – կիսասիմետրիկ ենթաբազմաձևությունների լոկալ կառուցվածքը, որն ունի մեկ ոչ զրոյական սինգուլյար կորության գլխավոր վեկտոր և ռեգուլյար կորության գլխավոր վեկտորների մեկ խումբ՝ կազմված երկու գծորեն անկախ վեկտորներից:

Առանցքային բառեր. ռիչչի–կիսասիմետրիկ ենթաբազմաձևություններ, կիսաէյնշտեյնյան ենթաբազմաձևություններ, ուղիղ արտադրյալներ, գլան, կոն:

V.A. MIRZOYAN, G.S. MACHKALYAN, A.R. NAZARYAN

ON RICCI-SEMI-SYMETRIC SUBMANIFOLDS CODIMENSIONALITY THREE WITH TWO GROUPS OF PRINCIPAL CURVATURE VECTORS

In Euclidean space, the local structure of normally flat Ricci-semi-symmetric submanifolds of codimensionality three with one non zero singular principal curvature vectors and only with one group of regular principal curvature vectors consisting of two linearly independent vectors is investigated.

Keywords: Ricci-semi-symmetric submanifolds, semi-Einstein submanifolds, direct products, cylinder, cone.

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A.O. BABAYAN, M.H. MOHAMMADI

THE MATHEMATICAL MODELLING OF THE GARNISSAGE FURNACE

A mathematical model for glass melting in the garnissage furnace is obtained. It is supposed that the considered domain has cylindrical symmetry and therefore the problem can be reduced to a two-dimensional model.

Keywords: Navier-Stokes equation, continuity equation, energy conservation law, Stefan condition, Dirichlet condition, boundary value problem.

In the paper, a mathematical model for the glass melting process in the electric garnissage [1] furnace is obtained. The schematic cut of the furnace is presented in Fig. 1.

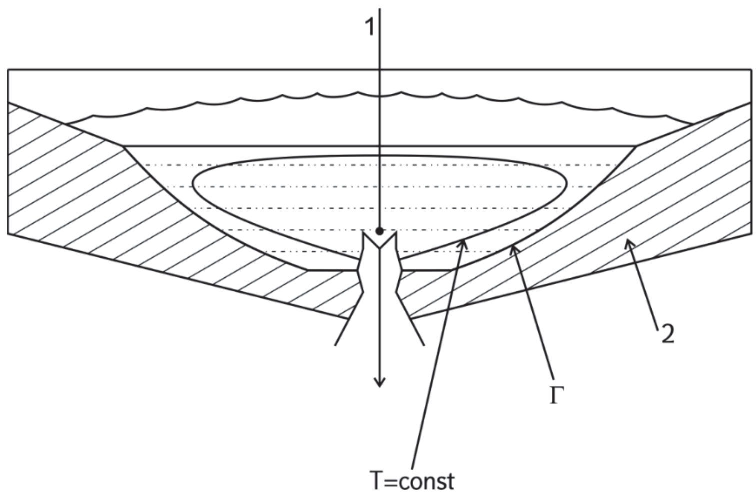

Fig. 1. The schematic cut of the garnissage furnace

In the center of the furnace (1), electrodes are located which create an electric arc – a source of high temperature required for glass melting. The melted glass passes through the molybdenum pipe to a hole at the bottom of the furnace. The temperature in the center of the furnace is extremely high but since the heat conduction coefficient is low, the exterior copper walls of the furnace are protected by a quickly cooling layer of solid glass - garnissage (2). The problem is to determine the free boundary Γ which separates the liquid phase of glass from the solid phase. Therefore, after determining Γ, it is possible to find out the optimal size of the furnace for the given capacity of the heat source (1).

The furnace used in production has cylindrical symmetry and thus we suppose that all processes take place in the two-dimensional domain, the schematic representation of which is presented in Fig. 2.

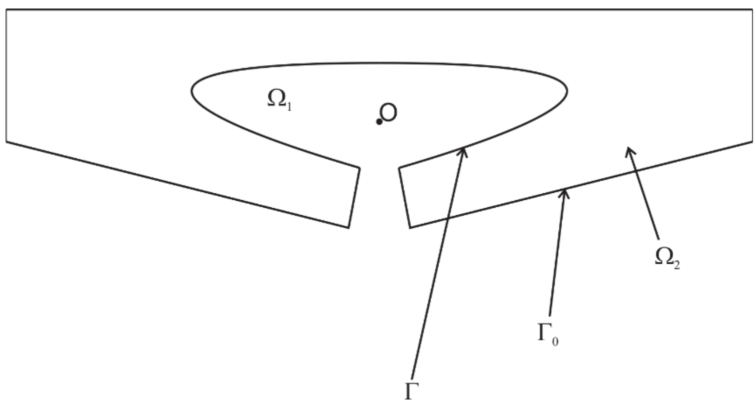

Fig. 2. The schematic representation of the process domain

Here, O is the heat source, Ω_1 - the liquid phase of the glass, Ω_2 - the solid phase, Γ is the free boundary, separating the liquid and solid phases and Γ_0 is the exterior boundary. In the domain Ω_1, the liquid mass is governed by general equations of hydrodynamics. We have to determine the main characteristics of the elemental mass of liquid glass: $\vec{V}$ - the velocity vector, p - the pressure and θ - the temperature in a fixed point. The first equation for determining these six unknown quantities is the continuity equation (mass conservation law). Supposing that the density ρ is a constant, this equation will have the form [2]:

$$div\left(\vec{V}\right)=0. \tag{1}$$

We must add to equation (1) two conservation laws: conservation of the linear momentum and conservation of energy. Supposing that the liquid mass is incompressible, we reduce the linear momentum conservation law [2] (Navier-Stokes equation) to the form:

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\bullet\nabla\right)\vec{V}=-\frac{1}{\rho}\nabla p+div\left(\vartheta\sigma\right)+\vec{g}, \tag{2}$$

Here $\vec{g}$ is the free fall acceleration, ϑ - the viscosity coefficient, $\sigma=\left(\sigma_{ij}\right)$ is a stress tensor, where

$$\sigma_{ij}=\frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right),\ i,j=1,2,3;\ \vec{V}=\left\{V_1,V_2,V_3\right\}. \tag{3}$$

The viscosity coefficient $\vartheta=\vartheta\left(\theta\right)$ depends on temperature θ, and, therefore, on the point coordinates. Supposing that this coefficient is constant and taking into account (1) we obtain the equation:

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\bullet\nabla\right)\vec{V}=-\frac{1}{\rho}\nabla p+\vartheta\Delta\vec{V}+\vec{g}. \tag{2M}$$

The energy conservation law, [3], is represented in the following form:

$$\frac{\partial \theta}{\partial t}+\left(\vec{V}\bullet\nabla\right)\theta=-\frac{k}{\rho c}\Delta\theta+\frac{1}{\rho c}F+2\vartheta\left|\sigma\right|^2, \tag{4}$$

where F is the heat source capacity, k is the thermal conductivity, c is the specific heat. We suppose, that k,c are constants. Thus, we get a system of five equations (1), (2), (4) (or (1), (2M), (4)) for the determination of five unknowns. For the unique determination of unknowns, we must add boundary conditions.

We consider a three-dimensional domain $G = (\Omega_1 \cup \Omega_2) \times \{t | t \in (0, M)\}$ with a smooth boundary and suppose that the unknown boundary Γ which separates the solid and liquid phases is determined by equation $\Phi(r, z, t) = 0$. In the solid phase $\vec{V} = 0$, therefore, in the domain $G_2 = \Omega_2 \times \{t | t \in (0, M)\}$, equation (5) is reduced to the heat conduction equation

$$\frac{\partial \theta}{\partial t} = -\frac{k}{\rho c} \Delta \theta + \frac{1}{\rho c} F$$

with boundary conditions

$$\theta|_{t=0} = \varphi(x_1, x_2, x_3),\ (x_1, x_2, x_3) \in \Omega_2\ ;\ \theta|_{\Gamma_0 \times (0, M)} = T_1,\ \theta|_{\Gamma} = T_2, \qquad (5)$$

where φ is the starting temperature distribution, T_1, T_2 are the prescribed constants so that T_2 is the temperature on the solid-liquid boundary. In the liquid phase, we have Dirichlet condition for $\theta = 0$

$$\theta|_{t=0} = \psi(x_1, x_2, x_3),\ (x_1, x_2, x_3) \in \Omega_1 \qquad (6)$$

and the Stefan condition on the phase-changing boundary:

$$L \frac{\partial \Phi}{\partial t} = -\left(\nabla \vec{V}(P-0) \bullet \nabla \Phi\right). \qquad (7)$$

Here L is the latent heat necessary for the phase changing, $\nabla \vec{V}(P-0)$ is the limit of $\nabla \vec{V}$ in the point $P \in \Gamma$ from the liquid phase. We must mention, that for amorphous substances (as, for example, glass) there is no phase change during solidification ([4], page 52). In this case $L = 0$, and on the free boundary, we have the condition:

$$\left(\nabla \vec{V}(P-0) \bullet \nabla \Phi\right) = 0. \qquad (8)$$

Finally, to determine the five unknown quantities $\vec{V}, p, \theta$, we get the system (1), (2), (4) (or (1), (2M), (4)) with boundary conditions (5), (6), (7), (or (5), (6), (8)). The case of the stationary problem (1), (2), (4), (5), (6), (7) (if t tends to infinity) has been investigated in [5].

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Ա.Հ. ԲԱԲԱՅԱՆ, Մ.Հ. ՄՈՀԱՄՄԱԴԻ

ԳԱՐՆԻՍԱԺ ՎԱՌԱՐԱՆԻ ՄԱԹԵՄԱՏԻԿԱԿԱՆ ՄՈԴԵԼԱՎՈՐՈՒՄԸ

Ստացվել է գարնիսաժ վառարանում ապակու հալման գործընթացի մաթեմատիկական մոդելը: Ենթադրվում է, որ դիտարկվող տիրույթը գլանաձև համաչափ է, ինչի հետևանքով խնդիրը հանգեցվում է երկչափ մոդելին:

Առանցքային բառեր. Նավյե-Ստոքսի հավասարումներ, անխզելիության հավասարում, էներգիայի պահպանման օրենք, Ստեֆանի պայման, Դիրիխլեի պայման, եզրային խնդիր:

А.О. БАБАЯН, М.Х. МОХАММАДИ

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ГАРНИСАЖНОЙ ПЕЧИ

Получена математическая модель процесса плавки стекла в гарнисажной печи. Предполагается, что рассматриваемая область обладает цилиндрической симметрией, и, следовательно, задача сводится к двумерной модели.

Ключевые слова: уравнения Навье–Стокса, уравнение неразрывности, закон сохранения энергии, условие Стефана, условие Дирихле, граничная задача.