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THE SPHERE WAVE IN THE FRAUNHOFER PICTURE

In the framework of the presented work we consider the problem of description of a superposition field generated by a system of point sources. It is suggested that a point source generates a sphere wave, which is considered in the Fraunhofer picture. In the general form, the problem of determining the maximums and minimums of the total wave field of many sources is discussed**.**

Keywords: diffraction theory, Fraunhofer picture.

Here, we investigate the superposition wave field generated by a system of point sources [1]:

$$U(\vec{R},t)=\sum_{j=1}^{N}\frac{a_0}{L_j}\cos\left[\omega t-kL_j+\gamma_j\right]\ , \tag{1}$$

where N is the number of the point sources, a_0 - the sphere wave amplitude, and the space vector $\vec{R}$ shows the observation point. Here L_j is the distance between the observation point and the j - source:

$$L_j=\sqrt{(z-z_j)^2+(x-x_j)^2+(y-y_j)^2}\ , \tag{2}$$

where x,y,z are the coordinates of an observation point and x_j,y_j,z_j are the corresponding coordinates of the j - source.

In the expressions of the space-dependent parts of phases of the sphere waves generated by single sources (see (1)), i.e. for the expressions

$$\varphi_j(\vec{R})=kL_j-\gamma_j\ \ (j=1,2,\cdots,N) \tag{3}$$

we took into account the fact that the wave field generation by different sources can differ by the phases at the initial time moment. So, the quantities γ_j are the initial magnitudes of the phases.

It is well known that if the problem is considered under the conditions

$$|x|,|y|<<L\ ,\ |x_j|,|y_j|,|z_j|<<L\ , \tag{4}$$

when all the amplitudes of sphere waves presented in the superposition field expression (1) take the same value: (see, for example, [1])

$$\frac{A_0}{R_j}=\frac{A_0}{\sqrt{(z-z_j)^2+(x-x_j)^2+(y-y_j)^2}}\approx\frac{A_0}{L}\ , \tag{5}$$

where $L=|z|$. Using (3) and (5), the sum (1) can be written as:

$$U(\vec{R},t)=\frac{a_0}{L}\sum_{j=1}^{N}\cos\left[\omega t-\varphi_j(\vec{R})\right]. \tag{6}$$

Let us consider the points where the following conditions take place:

$$\varphi_1(\vec{R})=\pi n_1\ ,\ \varphi_2(\vec{R})=\pi n_2\ ,\ \cdots,\ \varphi_N(\vec{R})=\pi n_N\ , \tag{7}$$

where $n_j=0,1,2,\cdots$. We will call these points the phase points. It is easy to see that at the phase points, the expression of the general field (6) takes the form of:

$$U(\vec{R},t)=A_0\sum_{j=1}^{N}(-1)^{n_j}\cos(\omega t), \tag{8}$$

where $A_0=a_0/L$. From this formula it follows that if all n_j are even or if all n_j are odd, the oscillations occur with the maximum amplitude;

$$U(\vec{R},t)=\pm NA_0\cos(\omega t)\,. \tag{9}$$

It should be noted that the oscillations, when all n_j are even, and the oscillations when all n_j are anti-phase to each other, the phase points at which the oscillations occur with the maximum possible amplitude are usually called the points of the main maxima.

It is obvious, that if in (8), only one of n_j is an odd number and all the others are even numbers, or vice versa (only one of n_j in (8) is an even number and all the others are odd numbers), then the oscillation at such points takes the form:

$$U(\vec{R},t)=\pm(N-1)A_0\cos(\omega t)\,. \tag{10}$$

These points can be called the maxima of the $N-1$ order.

Guided by the same logic, it is easy to see that the field amplitude or the order of the maximum at the phase point is determined by the difference between the numbers of the even and odd n_j in (8). Let N' be the number of sources for which n_j are odd (even). Then, the number of sources for which n_j will be an even (odd) number is $N-N'$. It is clear that in this case the field will look like:

$$U(\vec{R},t) = \pm(N - N')A_0 \cos\left(\omega t\right). \tag{11}$$

Note that this expression will correspond to the maximum of the $N - N'$ order.

As mentioned above, the quantities γ_j are the initial magnitudes of the phases $\varphi_j(\vec{R}) = kL_j - \gamma_j$ (see (3)). Let the radiation of sources be induced by the presence of some primary field $\psi(\vec{r},t)$ in space. This field is an external mechanism for creating forces for all sources, under the action of which the sources begin to generate waves. It is clear that, if we neglect the influence of the secondary field created by the sources on the sources themselves, then the oscillations of the sources will be determined only by the magnitude of the primary field at the points of the sources. For example, in the diffraction of X-rays on a crystal lattice, when the primary X-ray penetrating into the volume of the crystal leads to the excitation of secondary waves on the atoms of the lattice. As it is known, in this case, the difference in wave paths from different atoms is determined not only by their mutual position, but also by the directionality of the primary beam [2-5].

Let the primary field $\psi(\vec{r},t)$ have the form of a plane wave;

$$\psi(\vec{r},t) = B_0 \cos(\omega t - \vec{K}\vec{R}), \tag{12}$$

where the quantity B_0 is its amplitude and

$$\vec{K} = k \cdot \vec{e}, \tag{13}$$

where $\vec{e}$ is the unit dimensionless vector showing the propagation derestriction. In accordance with the above mentioned, in the dipole approximation (the spatial dependence of the primary field within a one-source origin can be neglected) the initial phases γ_j take the form:

$$\gamma_1 = \vec{K} \cdot \vec{r}_1, \ \gamma_2 = \vec{K} \cdot \vec{r}_2, \ \cdots, \ \gamma_N = \vec{K} \cdot \vec{r}_N. \tag{14}$$

Taking into account the connection between the wave number k and the wave length λ ($k = 2\pi / \lambda$) and using (13) for (14), one can get:

$$L_1 - \vec{e} \cdot \vec{r}_1 = \frac{\lambda}{2} n_1, \ L_2 - \vec{e} \cdot \vec{r}_2 = \frac{\lambda}{2} n_2, \ \cdots, \ L_N - \vec{e} \cdot \vec{r}_N = \frac{\lambda}{2} n_N. \tag{15}$$

The quantities $L_j - \vec{e} \cdot \vec{r}_j$ ($j = 1, 2 \cdots, N$) are the wave paths. As it follows from the conditions (7), if both n_j and n_i are even at the same time, or are simultaneously odd, then the waves from the j-th and i-th sources amplify each other. It is clear as well that for the same parity of n_j and n_i the difference $n_i - n_j$ is always an even number. So, if $n_i - n_j = 2n_{ij}$ ($n_{ij} = 0, \pm 1, \pm 2, \cdots$), i.e. n_j and n_i have the same parity, the path difference takes the form:

$$L_i - L_j - \vec{e}(\vec{r}_i - \vec{r}_j) = \lambda n_{ij}\,, \tag{16}$$

and the waves from these sources amplify each other. If n_j and n_i have a different parity, which means that $n_i - n_j = 2n_{ij} + 1$ ($n_{ij} = 0, \pm 1, \pm 2, \cdots$), the waves of the j-th and the i-th sources extinguish each other;

$$L_i - L_j - \vec{e}(\vec{r}_i - \vec{r}_j) = \lambda (n_{ij} + 1/2)\,. \tag{17}$$

Let us now assume that the generation of waves at the sources occurs with the same initial phases, i.e. in (16) and (17) all the products $\vec{e}(\vec{r}_i - \vec{r}_j) = 0$. It can be in the case the wave vector $\vec{K}$ (13) of the primary wave is perpendicular to all vectors $\vec{r}_j$ ($j = 1, 2 \cdots, N$) and, therefore, $\vec{e} \cdot \vec{r}_j = 0$. In this case, the formulas (16), (17) take the form:

$$L_i - L_j = \lambda n_{ij}\,, \tag{18}$$

$$L_i - L_j = \lambda (n_{ij} + 1/2)\,. \tag{19}$$

When $N = 2$, it is easy to see, that these conditions are nothing more than the known conditions of the maxima and minima of the interference pattern. In accordance with this, we will call relations (18), (19) , as well as (16), (17), as the conditions of maxima and minima for a system of N sources.

As it is shown in Ref. [6] for the Fraunhofer picture, when the following conditions take place:

$$\sqrt{\frac{k}{2\pi L}}\left|x_j\right|, \sqrt{\frac{k}{2\pi L}}\left|y_j\right| << 1, \tag{20}$$

the sum (6) takes the form:

$$U(\vec{R}, t) = \frac{a_0}{L} \sum_{j=1}^{N} \cos\left[\omega t - k \cdot z_j - \frac{x \cdot x_j + y \cdot y_j}{L} k - \gamma_j\right]. \tag{21}$$

In the general case of arbitrary arranged sources, this expression can be considered by numerical methods only. However, if the location of sources has a symmetry, or a certain regularity is presented in it, when on the basis of formula (21) analytic results can be found. So, let us suppose that all the sources locate on the plate $z=0$ and the initial phases of all equal zero;

$$z_j = 0 \text{ and } \gamma_j = 0 \ (j = 1,2,\cdots,N) . \tag{22}$$

In this case, sum (21) takes the form:

$$U(\vec{R},t) = \frac{a_0}{L}\sum_{j=1}^{N}\cos\left[\omega t - \frac{x\cdot x_j + y\cdot y_j}{L}k\right] . \tag{23}$$

In the general case of arbitrary arranged sources, this expression of the superposition field written for Fresnel can be considered by numerical methods only. However if the location of sources has a symmetry, or a regularity is presented in it, when on the basis of (23) analytic results can be obtained. So let us suppose that the sources located on the plane (X,Y) are arranged in this plane on a certain circle centered at the origin of the coordinate system. It means that for the coordinates of all sources it takes place:

$$x_j^2 + y_j^2 = d^2 , (j = 1,2,\cdots,N) \tag{24}$$

where the quantity d is the radius of the above mentioned circle. Introducing the polar angle α on the plane (X,Y) the coordinates of the sources can be presented as:

$$x_j = d\cos\alpha_j , \ y_j = d\sin\alpha_j , \tag{25}$$

where α_j is the polar angle of the j-th source. If the location of the sources on the circle is unformal, it is easy to see that the dependence of α_j on its index has the form:

$$\alpha_j = \Delta\alpha(j-1) , \tag{26}$$

where

$$\Delta\alpha = \frac{2\pi}{N} \tag{27}$$

and we take $\alpha_1 = 0$. Now, let us present the Cartesian coordinates of the observation point by means of polar coordinates:

$$x = \rho\cos\alpha , \ y = \rho\sin\alpha . \tag{28}$$

Using (25) and (28), the sum (23) can be written as:

$$U(\vec{R},t)=\frac{a_0}{L}\sum_{j=1}^{N}\cos\left[\omega t-\frac{\rho d}{L}k\cos(\alpha-\alpha_j)\right]\ . \tag{29}$$

It is easy to see, that for the case of the periodic structure, this sum gives the well- known result of the diffraction theory.

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Ա.Ժ. ԽԱՉԱՏՐՅԱՆ

ՍՖԵՐԻԿ ԱԼԻՔԸ ՖՐԱՈՒՆՀՈՖԵՐԻ ՊԱՏԿԵՐՈՒՄ

Աշխատանքի շրջանակներում դիտարկում է կետային աղբյուրների համակարգի կողմի գեներացրած սուպերպոզիցիոն դաշտի նկարագրման խնդիրը: Մեկ աղբյուրից առաջացած գնդային ալիքը դիտարկվում է Ֆրաունհոֆերի պատկերում: Ընդհանուր ձևով քննարկվում է բազմաթիվ աղբյուրների ընդհանուր ալիքային դաշտի ինտենսիվության առավելագույնի և նվազագույնի որոշման խնդիրը:

Առանցքային բառեր. դիֆրակցիայի տեսություն, Ֆրաունհոֆերի պատկեր:

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СФЕРИЧЕСКАЯ ВОЛНА В КАРТИНЕ ФРАУНГОФЕРА

В рамках представленной работы рассматривается задача описания поля, генерированного системой точечных источников. Сферическая волна одиночного источника рассматривается в картине Фраунгофера. В наиболее общем виде обсуждается задача определения максимумов и минимумов поля системы источников.

Ключевые слова: теория дифракции, картина Фраунгофера.