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STABILITY ANALYSIS OF A UAV EQUIPPED WITH A ROBOTIC ARM

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The paper presents a detailed stability analysis of an unmanned aerial vehicle (UAV) equipped with a 2-DOF robotic arm designed for grasping and manipulating different payloads. UAV-manipulators have got increasing attention due to their ability to perform complex manipulation tasks while flying. Despite the advantages, these systems face various challenges, and their control strategies need to be examined to ensure reliable and stable performance in such dynamic conditions. To address these challenges, the dynamics of the UAV-manipulator system are discussed in the paper. The manipulator's kinematics are modeled according to the Denavit-Hartenberg convention. The full system dynamics are derived using the Euler-Lagrange method, considering both UAV and robotic arm dynamics with their coupled interactions. A key contribution of this work is the formulation of the inertia matrix of the system as a constant nominal component and a time-varying uncertainty as a result of the effects of manipulator motion and object interaction. This separation enables a more structured and detailed analysis of the system's stability under changing robot configurations and payloads. To support the analysis, a Matlab/Simulink model of the UAV-manipulator system is developed based on the derived symbolic dynamics. The simulation model allows to test various manipulation scenarios and helps to monitor the key indicators of stability such as the system's response, center-of-mass behavior, and the effects of payload variations on the system's stability. The results emphasize the impact of the manipulator's configuration and the payload on the overall dynamics of the system. The proposed framework serves as a foundation for the future development of adaptive or data-driven control strategies that can address the dynamic uncertainties of the UAV-manipulator system.

Keywords: unmanned aerial vehicle, robotic manipulation, object grasping, stability analysis, Matlab/Simulink.

Introduction. UAVs equipped with robotic maipulators commonly referred to as aerial manipulators, have become a very popular research topic in the last decade. By combining aerial mobility with the ability of physical interaction with the world, these systems are offering a solution for hard-to-reach environment tasks. The aerial manipulators are used in various fields like military applications, photography and inspection, transportation, architecture, building, and construction [1].

However, aerial manipulators introduce a lot of challenges in terms of system modelling and control. A significant body of research has developed models for UAVs with robotic arms, focusing on areas such as stability analysis, control design, and dynamic interaction between the UAV and manipulator. These models often use Euler-Lagrange or Newton-Euler methods to integrate the dynamics of both components into a single system [2-4]. Other works model the manipulator as an external disturbance acting on the UAV, enabling relatively simplified control design, considering the robot arm movements as a disturbance and compensating with a disturbance observer controller [5].

As the manipulator moves or interacts with objects, the UAV's center-of-mass changes, which can significantly affect its stability and control. To minimize the impact of the manipulator's movements on the UAV, a multilayer control system is proposed in [6], which incorporates a moving battery mechanism to counteract the center-of-mass displacement caused by the arm's motion, effectively keeping the UAV's center-of-mass as close as possible to its geometric center during operation. In [7] a generalized CoG a compensation scheme is implemented, where online estimation of the center-of-mass allows for compensation of position drift, thereby improving the UAV's stability and manipulation accuracy.

In the paper, the UAV and manipulator are modeled as a unified system to accurately capture all dynamic coupling effects, including changes in mass distribution and inertia during manipulation. To analyse the stability, we further decompose the system's inertia matrix into a constant nominal component and a time-varying uncertainty. This structure allows a more tractable analysis of how the manipulator motion and payload interaction influence the system’s stability, without compromising the fidelity of the underlying dynamic model. The approach is a foundation for both detailed system analysis and future development of adaptive or disturbance-aware control strategies.

System modeling. This section describes the complete modeling framework of the aerial manipulator system, which consists of a quadrotor UAV equipped with a 2-DOF robotic arm mounted on its underside (Fig. 1). The modeling approach captures the coupled dynamics between the UAV and the manipulator using a unified formulation.

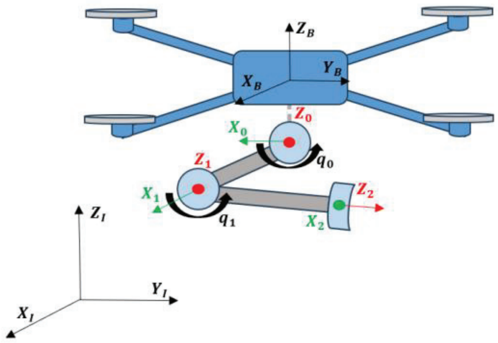

Fig. 1. An UAV-manipulator system in the initial (folded) state

To describe the system, we have used two frames: the inertial frame *{I}* and the body fixed frame *{B}*. The transformation between them is given by the rotation R_{IB} matrix, constructed from the UAV's Euler angles (roll, pitch, yaw) using a ZYX convention. The UAV is modeled as a 6-DOF rigid body: with 3 translational coordinates and 3 rotational angles. The manipulator consists of two revolute joints that operate in the vertical plane beneath the UAV. The base of the manipulator is assumed to be rigidly fixed to the downside of the UAV, and the base frame of the manipulator is aligned with the UAV's body-fixed frame.

The combined configuration of the system is described by the generalized coordinate vector:

$$q = [x, y, z, \phi, \theta, \psi, \varepsilon_1, \varepsilon_2]^T,$$

where $\xi = [x, y, z]^T$ represents the UAV's position in inertial coordinates, $\Omega = [\phi, \theta, \psi]^T$ denotes the UAV's Euler angles, and $\Upsilon = [\varepsilon_1, \varepsilon_2]^T$ are the joint variables of the robotic arm.

The linear and angular velocities of the UAV in the inertial frame can be expressed as:

$$\dot{P}_{UAV} = R_{IB} P^{B}_{UAV}, \quad (1)$$

$$\omega^{B}_{UAV} = R_{IB}{}^{T} T \dot{\Omega}, \quad (2)$$

where the angular velocity is expressed via Euler angles using the standart transformation matrix T [8, 9].

In order to present the configuration of the robotic arm, we have used the Denavit–Hartenberg (DH) convention, which provides a standardized methodology for defining the relative transformations between consecutive links [10]. To formulate the manipulator's kinetic energy, we computed the linear and angular velocities of each link’s center-of-mass:

$$\dot{p}_i = \dot{P}_{UAV} + S(\omega)R_{IB}p_i + R_{IB}J_p(\Upsilon)\dot{\Upsilon}, \quad (3)$$

$$\dot{\omega}_i = \omega + R_{IB}J_\omega(\Upsilon)\dot{\Upsilon}, \quad (4)$$

where $S(\omega)$ is the skew-symmetric matrix, J_p and J_ω are the Jacobian matrices that relate the generalized velocity $\dot{\Upsilon}$ to the velocities of the i-th link’s center-of-mass. These Jacobians are derived by differentiating the kinematic chain transformations with respect to Υ, and include the influence of both UAV base motion and the joint variables of the arm.

Using the kinematic relationships (1)-(4), energy expressions are derived which form the basis of the Euler–Lagrange equations. The total kinetic energy E of the UAV-manipulator system is the sum of the UAV’s kinetic energy and the kinetic energies of the two arm links:

$$E(q,\dot{q}) = E_{UAV} + E_{arm,1} + E_{arm,2}. \quad (5)$$

Modeled as a rigid body of mass m_{UAV} and inertia tensor I_{UAV} for UAV and m_i and I_i for each manipulator’s links we have:

$$E_{UAV} = \frac{1}{2}m_{UAV}\dot{P}_{UAV}{}^T\dot{P}_{UAV} + \frac{1}{2}\dot{\Omega}^T T R_{IB} I_{UAV} R_{IB}{}^T T^T \dot{\Omega}, \quad (6)$$

$$E_{arm,i} = \frac{1}{2}m_i\dot{p}_i{}^T\dot{p}_i + \frac{1}{2}\dot{\omega}_i{}^T R_{IB} I_i R_{IB}{}^T \dot{\omega}_i. \quad (7)$$

The gravitational potential energy U comprises contributions from the UAV and each link:

$$U(q) = m_{UAV}gz + \sum_{i=1}^{2} m_i g z_i(q), \quad (8)$$

where z is the UAV’s altitude and $z_i(q)$ is the vertical coordinate of link i’s center-of-mass in the inertial frame, and g is the gravitational acceleration.

The total kinetic and potential energy of the UAV-manipulator system is computed based on the symbolic kinematic model. Defining the Lagrangian:

$$L(q,\dot{q}) = E(q,\dot{q}) - U(q), \quad (9)$$

the Euler–Lagrange equations:

$$\frac{d}{dt}\frac{\delta L}{\delta \dot{q}} - \frac{\delta L}{\delta q} = Q, \quad (10)$$

yield the system’s equamtions of motion. Here $Q = [\tau^T + \tau_D{}^T]^T$ collects the generalized control inputs τ and disturbance forces τ_D.

The resulting equations of motion take the standard form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_D, \quad (11)$$

where $M(q)$ is R^{8x8} symmetric, positive-definite inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ is the gravity vector.

Stability analysis. To evaluate the dynamic behavior and stability of the UAV-manipulator system under different operating conditions, a comprehensive simulation framework is developed in Matlab/Simulink. This framework incorporates the full nonlinear dynamics and allows systematic tracking of stability indicators such as inertia variations, center-of-mass drift, and system responses during manipulation and object grasping.

A key point of the paper is the analysis of stability by separation of the $M(q)$ inertia matrix as a composition of a constant nominal inertia matrix M_0, representing the configuration at the initial (folded) state (Fig. 1), and a time-varying uncertainty $\Delta M(q)$ changing from the manipulator's motion and interaction with payloads. The total inertia is thus expressed as:

$$M(q) = M_0\big(I + \Delta(q)\big), \tag{12}$$

where $\Delta(q) = {M_0}^{-1}\Delta M(q)$ defines a multiplicative uncertainty model that captures how the inertia varies are relative to its nominal structure (Fig. 2).

Through this decomposition, the stability analysis becomes a study of the system's robustness to structured dynamic perturbations. If the gain of the uncertainty channel $\Delta(q)$ remains sufficiently small, and the nominal system ${M_0}^{-1}(\tau - C\dot{q} - G)$ is input-to-state stable, then the full system is guaranteed to remain stable despite changing the manipulator configurations and payload effects [11].

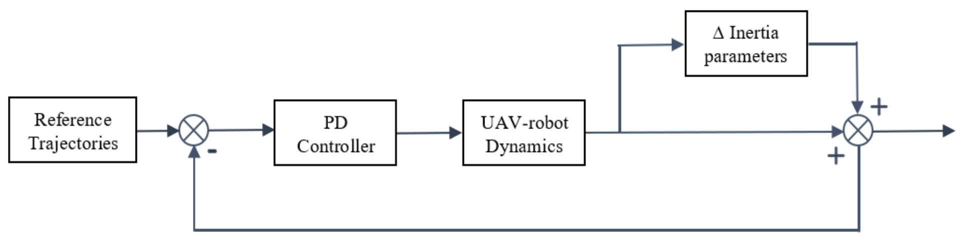

Fig. 2. Inertia changes modelled as multiplicative uncertainty

To investigate the stability of the system, we have conducted a simulation where the UAV maintains a hovering state while the manipulator transitions from a folded to an extended configuration (Fig. 3). The initial folded state is chosen to reduce center-of-mass displacement and dynamic coupling at takeoff. As the joints extend over time, at the 5th second, the manipulator grasps the object, introducing a dynamic shift in the system's inertia. A PD controller was chosen and designed for the system's nominal inertia matrix to stabilize the system.

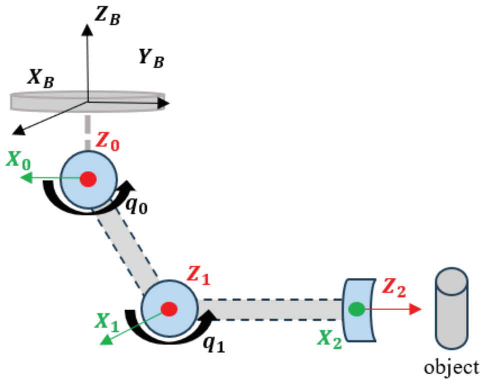

Fig. 3. Object grasping scenario

To quantify the effects of these dynamic changes, the infinite norm $\|\Delta M(q)\|_\infty$ of the changing component of the inertia matrix is tracked throughout the simulation. This metrix provides a direct measure of the system's deviation from its nominal dynamics.

The numerical values listed in the Table below are used for simulating the UAV and manipulator system.

Table

Numerical values for simulation

UAV mass	m_{UAV}	2.4	*kg*
UAV dimensions	dim_{UAV}	0.5x0.5x0.2	*m*
Link masses	m_1, m_2	0.3, 0.45	*kg*
Link dimensions	dim_{L_1}, dim_{L_2}	0.05x0.05x0.2, 0.05x0.05x0.3	*m*
Grasped object mass	m_{obj}	0.5	*kg*
Grasped object dimensions	dim_{obj}	0.08x0.08x0.15	*m*

Two surface plots are presented to visualize how $\|\Delta M(q)\|_\infty$ evolves across the full range of joints configurations, first in the case of a free-moving manipulator, and second when the object is grasped by the end-effector (Fig. 4). These figures highlight the dynamic variation due to payload interaction.

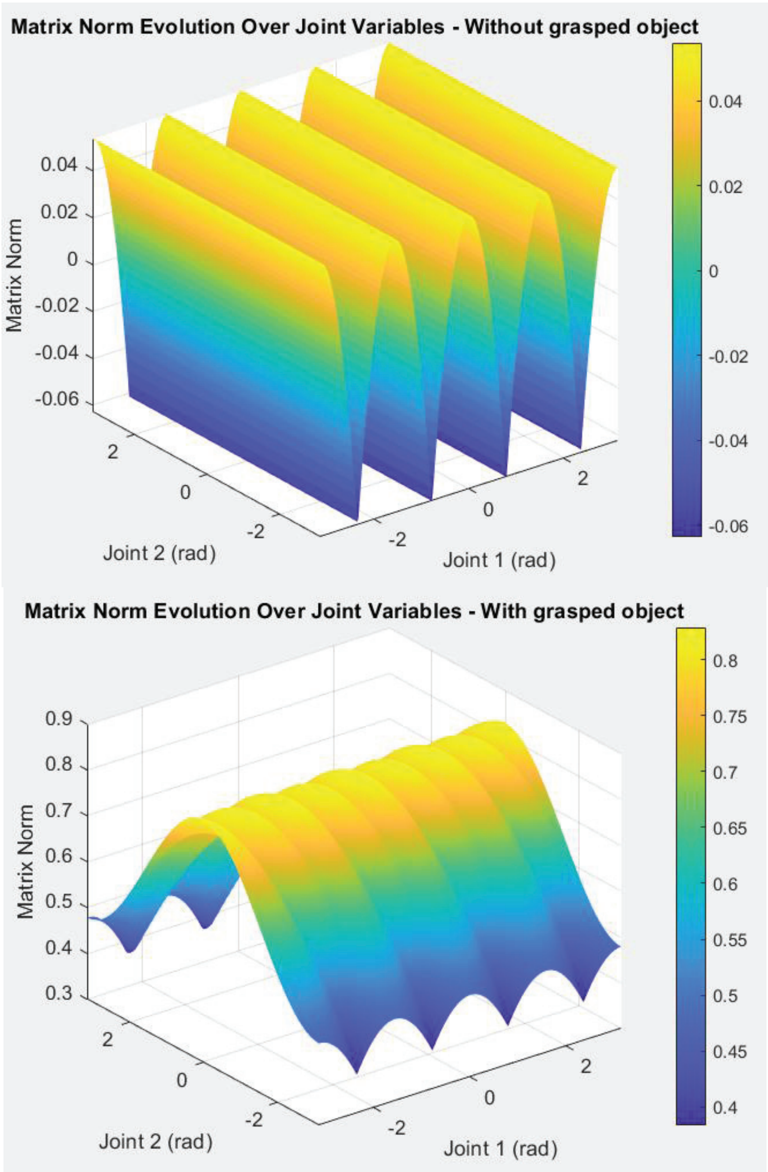

Fig. 4. The effect of joint configurations on inertia changes

Similarly, the displacement of the system's center-of-mass relative to the UAV base frame center-of-mass is monitored to assess the physical impact of inertia variation on the system's balance and stability (Fig. 5).

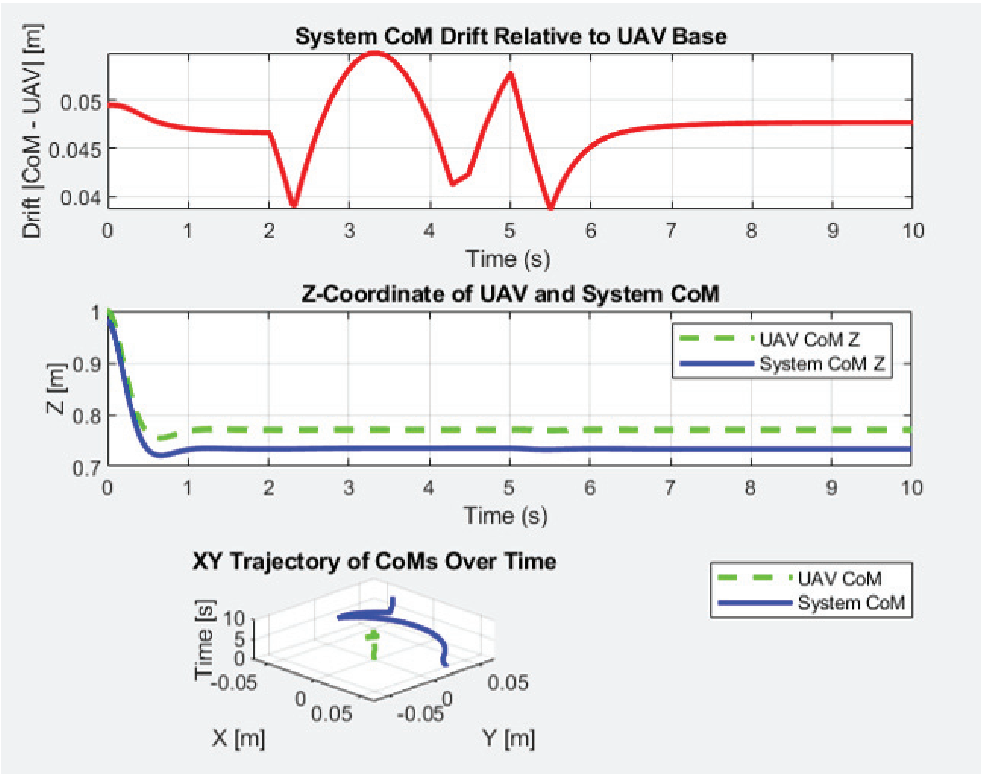

Fig. 5. The system center-of-mass drift relative to the UAV base

In addition to tracking the system's center-of-mass behavior and inertia variation, further simulations are conducted to analyze the system's response along the X, Y axes and orientation angles with the nominal constant inertia matrix and with the full changing inertia matrix that includes the manipulator configuration and object grasping (Fig. 6).

The resulting plots reveal that the system driven by the nominal inertia model shows smoother and more stable responses, with minor oscillations and quicker convergence. In contrast, the configuration including the full dynamic inertia demonstrates increased disturbances, especially following the grasping events, where the inertia shift introduces nonlinear effects. This contrast illustrates the sensitivity of the UAV-manipulator system to inertia uncertainty and shows the importance of incorporating such effects in any robust control design. The nominal model, while simplified, provides a stable baseline for initial operation, however, it is not able to capture the changing dynamics introduced by payload interaction, making clear the necessity for adaptive or uncertainty-aware control strategies in future developments.

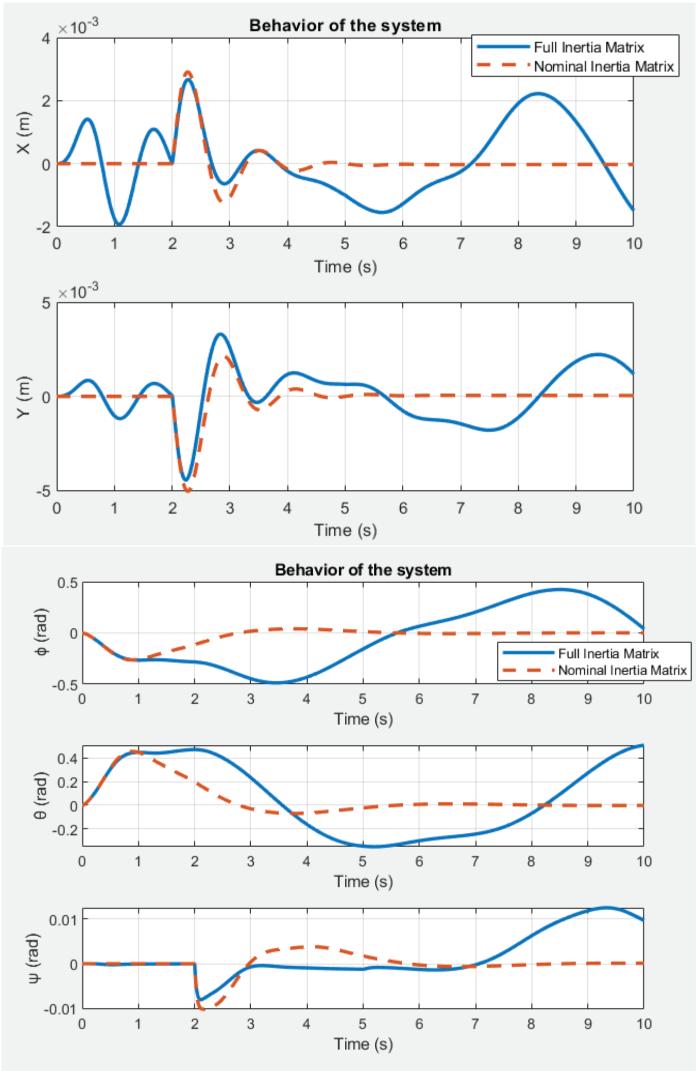

Fig. 6. The UAV response under nominal vs. full inertia

Conclusion. The paper presents a structured stability analysis of an UAV equipped with a 2-DOF robotic manipulator. The full nonlinear dynamics of the coupled system are derived using the Euler–Lagrange method, capturing both the UAV and the manipulator arm, including payload interaction. A key contribution of this work is the modeling of the inertia matrix as a nominal component and a time-varying perturbation, enabling a direct measure of how arm motion and grasped payloads affect system stability. A simulation framework is developed in Matlab/Simulink to evaluate the behavior of the system during manipulation tasks. The results indicate that although the system remains dynamically bounded, the observed variations in the inertia profile and center-of-mass position introduce measurable disturbances. These effects remain within acceptable limits under the tested conditions, however, to achieve improved stability and disturbance rejection in more demanding tasks, additional methods such as adaptive or data-driven control should be employed, which can respond to real-time changes in the inertia model.

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References

1. **Meng J., Buzzatto J., Liu Y., and Liarokapis M.** On aerial robots with grasping and perching capabilities: A Comprehensive Review // Frontiers in Robotics and AI.- Mar. 2022.-Vol. 8.- P. 1-24.
2. **Lippiello V., and Ruggiero F.** Exploiting redundancy in cartesian impedance control of UAVS equipped with a robotic arm // Proc. IEEE Int. Conf. Intell. Robots. Syst.- Oct. 2012.- P. 3768-3773.
3. Adaptive Robust Controller Design-Based RBF Neural Network for Aerial Robot Arm Model / **I. Al-Darraji, et al //** Electronics.- 2021.- 10.- P. 831.
4. **Yilmaz E., Zaki H., and Unel M.** Nonlinear Adaptive Control of an Aerial Manipulation System // 18th European Control Conference (ECC).- Naples, Italy, 2019.- P. 3916-3921.
5. Robust Control for Unmanned Aerial Manipulator Under Disturbances / **Y. Chen, et al //** IEEE Access.- 2020.- Vol. 8.- P. 129869–129877.
6. A multilayer control for multirotor uavs equipped with a servo robot arm, in Robotics and Automation (ICRA) / **F. Ruggiero, et al** // IEEE International Conference on. IEEE.- 2015.- P. 4014–4020.
7. **Fresk E., Wuthier D., and Nikolakopoulos G.** Generalized center of gravity compensation for multirotors with application to aerial manipulation // Proc. IEEE Int. Conf. Intell. Robots. Syst.- Sep. 2017.- P. 4424-4429.
8. **Li D., Hongtao W.** Dynamical Modelling and Robust Control for an Unmanned Aerial Robot Using Hexarotor with 2-DOF Manipulator // International Journal of Aerospace Engineering.- 2019.- ID 5483073.- P. 12.
9. **Seyyed Ali E., Afshin B** Simultaneous trajectory tracking and aerial manipulation using a multi-stage model predictive control // Aerospace Science and Technology.- 2021.- Vol. 112.- P. 19.
10. **Spong M.W., Hutchinson S., Vidyasagar M.** Robot Modeling and Control.- USA, John Wiley & Sons, Inc, 2006.- 407p.
11. **Bosgra O.H., Miele A., Luyben W.L.** Design Methods For Control Systems.- John Wiley & Sons, Inc, 2002.- 319p.

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ՌՈԲՈՏԱՅԻՆ ՁԵՌՔՈՎ ՕԺՏՎԱԾ ԱԹՍ-Ի ԿԱՅՈՒՆՈՒԹՅԱՆ ՎԵՐԼՈՒԾՈՒԹՅՈՒՆ

Տ.Ա. Սիմոնյան

Ներկայացված է անօդաչու թռչող սարքի (ԱԹՍ) կայունության մանրամասն վերլուծություն, որն օժտված է 2 ազատության աստիճանով ռոբոտային ձեռքով և նախատեսված է տարբեր օգտակար բեռներ բռնելու և տեղափոխելու համար: ԱԹՍ-ռոբոտները մեծ ուշադրության են արժանանում թռիչքի ընթացքում բարդ մանիպուլյատիվ առաջադրանքներ կատարելու ունակության շնորհիվ: Չնայած առավելություններին՝ այս համակարգերը բախվում են տարբեր մարտահրավերների, և անհրաժեշտ է ուսումնասիրել դրանց կառավարման առանձնահատկությունները՝ դինամիկ պայմաններում հուսալի և կայուն վարքագիծ ապահովելու համար: Այդ խնդիրները լուծելու նպատակով աշխատանքում դիտարկվում է ԱԹՍ-մանիպուլյատոր համակարգի դինամիկան: Մանիպուլյատորի կինեմատիկան մոդելավորվել է Դենավիթ-Հարթենբերգի կոնվենցիայի համաձայն: Համակարգի ամբողջական դինամիկան ստացվել է Էյլեր-Լագրանժի մոդելավորման մեթոդների կիրառմամբ, որոնք հաշվի են առնում ինչպես ԱԹՍ-ի, այնպես էլ ռոբոտ-մանիպուլյատորի դինամիկան՝ իրենց փոխկապակցված փոխազդեցություններով: Աշխատանքի հիմնական ներդրումն է համակարգի իներցիայի մատրիցի ձևակերպումը՝ որպես հաստատուն բաղադրիչ և ժամանակի ընթացքում փոփոխվող անորոշություն՝ ռոբոտային ձեռքի շարժման և օբյեկտների հետ փոխազդեցության հետևանքով: Այս տարանջատումը հնարավորություն է տալիս՝ իրականացնելու համակարգի կայունության ավելի կառուցվածքային և մանրամասն վերլուծություն ռոբոտի կոնֆիգուրացիաների և օգտակար բեռների փոփոխման պայմաններում: Վերլուծությունն իրականացնելու նպատակով ստեղծվել է ԱԹՍ-մանիպուլյատոր համակարգի Matlab/Simulink մոդելը՝ ստացված սիմվոլիկ դինամիկայի հիման վրա: Սիմուլյացիոն մոդելը թույլ է տալիս փորձարկել տարբեր սցենարներ և օգնում է վերլուծել կայունության հիմնական ցուցանիշները, ինչպիսիք են համակարգի արձագանքը, զանգվածների կենտրոնի վարքը և բեռի փոփոխությունների ազդեցությունը համակարգի կայունության վրա: Արդյունքները կարևորում են մանիպուլյատորի կոնֆիգուրացիայի և բեռի ազդեցությունը համակարգի ընդհանուր դինամիկայի վրա: Աշխատանքը հիմք է հանդիսանում հարմարվողական կամ տվյալների վրա հիմնված կառավարման համակարգի հետագա

մշակման համար, որոնք հաշվի կառնեն ԱԹՍ-մանիպուլյատոր համակարգի դինամիկ անորոշությունները:

Առանցքային բառեր. անօդաչու թռչող սարք, ռոբոտային մանիպուլյացիա, օբյեկտների բռնում, կայունության վերլուծություն, Matlab/Simulink:

АНАЛИЗ УСТОЙЧИВОСТИ БЕСПИЛОТНОГО ЛЕТАТЕЛЬНОГО АППАРАТА, ОСНАЩЕННОГО РОБОТИЗИРОВАННОЙ РУКОЙ

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Представлен подробный анализ устойчивости беспилотного летательного аппарата (БПЛА), оснащенного роботизированной рукой с 2-мя степенями свободы, предназначенной для захвата различных полезных грузов и манипулирования ими. БПЛА-манипуляторы привлекают все большее внимание благодаря своей способности выполнять сложные манипулятивные задачи во время полета. Несмотря на преимущества, эти системы сталкиваются с различными проблемами, и необходимо изучить их стратегии управления, чтобы обеспечить надежную и стабильную работу в таких динамичных условиях. Для решения этих задач в статье рассматривается динамика системы БПЛА-манипулятор. Кинематика манипулятора смоделирована в соответствии с конвенцией Денавита-Хартенберга. Полная динамика системы получена с использованием методов моделирования Эйлера-Лагранжа, учитывающих динамику как БПЛА, так и робота-манипулятора, с их связанными взаимодействиями. Ключевым вкладом работы является формулировка матрицы инерции системы как постоянного номинального компонента и изменяющейся во времени неопределенности, вызванной движением манипулятора и взаимодействием с объектом. Такое разделение позволяет проводить более структурированный и детальный анализ стабильности системы при изменении конфигурации робота и полезной нагрузки. Для поддержки анализа реализована модель системы БПЛА-манипулятор в среде Matlab/Simulink, основанная на полученной символьной динамике. Моделирование позволяет тестировать различные сценарии манипуляции и отслеживать ключевые показатели устойчивости, такие как реакция системы, поведение центра масс и влияние изменений полезной нагрузки на устойчивость системы. Предложенная структура служит основой для будущей разработки адаптивных систем управления или стратегий, основанных на данных, которые могут учитывать динамические неопределенности системы БПЛА-манипулятор.

Ключевые слова: беспилотный летательный аппарат, роботизированная манипуляция, захват объекта, анализ устойчивости, Matlab/Simulink.