ELECTRONICS

APPROXIMATE INVERSE DYNAMICS LINEARIZATION OF AN AERIAL MANIPULATOR SYSTEM

T.A. Simonyan

National Polytechnic University of Armenia

Aerial manipulators draw increasing attention due to their ability to achieve both aerial mobility and physical interaction with the world. With these capabilities, they offer solutions for tasks in hard-to-reach environments and are employed in various fields, including military applications, infrastructure inspection, construction and transportation. However, controlling these systems presents a lot of challenges due to their highly nonlinear, coupled, and time-varying dynamics. The existing nonlinear control methods, such as feedback linearization, require a perfectly known and continually updated dynamic model, which is computationally expensive and not always feasible for fast, real-world applications. This paper introduces a novel application of approximate inverse dynamics linearization to a quadrotor unmanned aerial vehicle (UAV) equipped with a 2 degree-of-freedom (DOF) robotic manipulator designed for grasping and manipulating different payloads. Our approach uses offline-computed, fixed nominal dynamics for the model linearization, treating the residual nonlinearities and inaccuracies as bounded structured uncertainties. This formulation simplifies the control problem, allowing for the design of robust controllers that can guarantee the stability and performance despite the dynamic changes of the system. Through numerical simulations, we demonstrate that this method achieves accurate trajectory tracking and effectively captures the influence of the manipulator motion on the system dynamics within the structured uncertainty. A key finding is the significant reduction in computational load, with approximate method being, on average, 50 times faster than online calculation of the full dynamics, proving its viability for real-time control applications.

Keywords: unmanned aerial vehicle, robot-manipulation, approximate inverse dynamics, linearization.

Introduction. The dynamics of an aerial manipulator combines the complex dynamics of the UAV with the robot-manipulators and represents a highly nonlinear and coupled nature. Such nonlinearities, variable parameters, and coupling effects make it inherently difficult to design a controller that will guarantee the stability and robustness to external disturbances. One of the key components in controlling such a system is to linearize the dynamics. Researchers have used various linearization techniques to achieve precise trajectory tracking and robust stability in

the presence of uncertainties and disturbances. In this section we review the recent literature, highlight their linearization methods and discuss the challenges and opportunities associated with each approach.

A common approach, particularly in early research or systems with limited computational power, is to linearize only the portion of the system dynamics, most often the UAV's altitude and position. This simplifies the control design but may neglect the dynamic effects of the manipulator.

In [1], an aerial gripper system is described using a custom-built quadrotor. The quadrotor's attitude is controlled using a PID controller based on the linearized dynamic model. They have linearized the rotational dynamics about the hover point using small angle approximations. This method is favoured for empirical tuning and demonstrates a pragmatic approach to linearization. Similarly, [2, 3] investigate stability control in aerial manipulation, where they develop control schemes to compensate for inertial changes. Their approach implies the use of an underlying, possibly simplified or linearized, model of the quadrotor's dynamics, which is then augmented with a mechanism to handle the manipulator's effect.

Adaptive control methods do not necessarily linearize the system's equations directly but are designed to make the closed-loop system behave like a desired linear reference model. They are particularly effective when the system parameters are unknown or change over time.

[4] proposes a Lyapunov-based MRAC scheme to achieve dynamic stability for an aerial vehicle with dual manipulators. The core idea is to design a controller that forces the nonlinear system to follow a stable, linear reference model. This method effectively "linearizes" the system's behavior without requiring perfect knowledge of its complex dynamics, as the controller adapts to unknown parameters. [5] uses a similar adaptive approach to control a UAV with varying payload, where adaptive laws are developed to handle full parametric uncertainties and make the system's behavior track the desired linear trajectory. [6] introduces an adaptive incremental nonlinear dynamic inversion (INDI) control scheme for aerial manipulators. INDI is a linearization-by-inversion technique that relies on real-time measurements to linearize the system dynamics incrementally, making it robust to model uncertainties. The adaptive part of their controller handles changes in inertia parameters caused by manipulator movements without needing explicit knowledge of the manipulator's dynamics. This approach linearizes the system's behavior by inverting its instantaneous dynamics based on measurements, thus avoiding the need for a perfect full-system model.

A number of papers apply feedback linearization to the combined UAV-manipulator system. [7] presents a detailed modeling and control approach for a

quadrotor with a manipulator, using a feedback linearization controller to manage the system. [8] also discusses the formulation of dynamics for aerial manipulators and the application of nonlinear control, with feedback linearization being a natural candidate for such a system. [9] explicitly mentions the use of a control algorithm based on the feedback linearization method and a PD regulator for a multi-rotor UAV equipped with a robotic manipulator, with a focus on compensating for changes in the inertia tensor and center of mass.

[10] applies feedback linearization to the nonlinear dynamics of the full aerial manipulator system. The paper models the dynamic coupling from the manipulator as a disturbance and then designs a robust H_∞ controller with a disturbance estimator to compensate for it. Similarly, [11] proposes a multi-stage model predictive control that includes a disturbance observer to handle the effects of model uncertainties and external disturbances on the full nonlinear system.

Those methods, while useful, have a complication as the need of having a perfectly known dynamic model, which should be changing according to the system configuration changes. This assumption is rarely true for real-word aerial manipulators. The coupled dynamics are complex, and mainly the ineartia matrix changes constantly as the manipulator moves or grasps an object. The real-time onboard computational load of the constantly changing full-system inverse dynamics is also a problem for such an agile system.

In this paper, we use the approximate inverse dynamics linearization approach, which, instead of assuming a perfect model, uses some nominal or estimated model for the linearization, treating the residual nonlinearities as bounded structured uncertainty. The concept of approximate inverse dynamics has been used to control robot-manipulators by [12-14]. In [13], the authors discuss the theoretical foundation of inverse dynamics and acknowledge the fact that in practice often the compensation can be imperfect, both for model uncertainty and for the approximations made in online computation of inverse dynamics, and introduce an additional term to an inverse dynamics controller which provides robustness to the control system by counteracting the effects of the approximations. This idea is further explored in [14], in which the authors apply the factorization approach to plant and controller models to characterize robustly stabilizing controllers for manipulators under approximate inverse dynamics control.

To the best of our knowledge, this is the first work to apply the approximate inverse dynamics linearization to the robust control of an aerial manipulator systems. In this paper, we provide a detailed analysis of this control method and application.

System modeling. This section describes the system model of the aerial manipulator system, which consists of a quadrotor UAV equipped with a 2-DOF robotic arm mounted on its underside. The modeling approach captures the coupled dynamics between the UAV and the manipulator using a unified formulation.

The UAV is modeled as a 6-DOF rigid body: with 3 translational coordinates and 3 rotational angles. The manipulato r consists of two revolute joints that operate in the vertical plane beneath the UAV. The base of the manipulator is assumed to be rigidly fixed to the downside of the UAV and the base frame of the manipulator is aligned with the UAV's body-fixed frame.

The combined configuration of the system is described by the controlled generalized coordinate vector:

$$q = [z, \phi, \theta, \psi, \varepsilon_1, \varepsilon_2]^T, \tag{1}$$

where z is the UAV's height in the inertial coordinates, $\Omega = [\phi, \theta, \psi]^T$ denotes the UAV's Euler angles, and $Y = [\varepsilon_1, \varepsilon_2]^T$ are the joint variables of the robotic arm. The UAV's x, y coordinates are controlled separately using the Ω vector.

The resulting equations of motion take the standard form [15, 16]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_D, \tag{2}$$

where $M(q)$ is R^{6x6} symmetric, positive-definite inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ is the gravity vector. The control impute $\tau = = [T_\Sigma, \tau_{UAV}, \tau_j]^T$ consists of quadrotor force and torques and manipulator joint torques:

$$\tau = [T_\Sigma, \tau_\phi, \tau_\theta, \tau_\psi, \tau_{j1}, \tau_{j2}]^T. \tag{3}$$

T_Σ is the total quadrotor thrust at hover ($T_\Sigma = \sum_{i=1}^4 T_i$), where each i th rotor generates a thrust T_i which is proportional to the square of angular velocity of rotors Ω_i (i.e. $T_i = c_T \Omega_i^2$, $c_T > 0$) and acts along the body-fixed axis z_B [17]. The $\tau_\phi, \tau_\theta, \tau_\psi$ are the quadrotor torques and τ_{j1}, τ_{j2} are the torques of each joint of the manipulator. The thrust to torque mapping for the quadrotor can be expressed using the 4-dimensional vector of thrusts T_i, ($\bar{T}_{UAV} = [T_1, T_2, T_3, T_4]^T$), and the quadrotor configuration matrix B_{UAV}:

$$\begin{bmatrix} T_\Sigma \\ \tau_{UAV} \end{bmatrix} = B_{UAV}\bar{T}_{UAV}. \tag{4}$$

Given the needed controls T_Σ and τ, the equation (4) allows computing the required thrusts T_i (or, which is equivalent, the velocities Ω_i) of rotors. The system's torque to input mapping can be described as:

$$\tau = B_{SYS}\bar{T}_{SYS}, \tag{5}$$

where $\bar{T}_{SYS}$ is the combined vector of the quadrotor motors thrusts and the manipulator joint torques. B_{SYS} for the X configuration quadrotor with 2-DOF robotic arm will have the form:

$$B_{SYS} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ \sqrt{2}L/2 & \sqrt{2}L/2 & -\sqrt{2}L/2 & -\sqrt{2}L/2 & 0 & 0 \\ -\sqrt{2}L/2 & \sqrt{2}L/2 & \sqrt{2}L/2 & -\sqrt{2}L/2 & 0 & 0 \\ -k_\psi & k_\psi & -k_\psi & k_\psi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{6}$$

where L is the length of the quadrotor arms, and k_ψ ($k_\psi > 0$) is the drag coefficient [18]. Looking at the determinant of the given matrix: $\det(B_{SYS}) = 8L^2 k_\psi$, it is obvious that it can be inverted for any given L and k_ψ.

Approximate inverse dynamics linearization. The reason behind applying the inverse dynamics linearization to the aerial manipulator system is to simplify a complex nonlinear dynamic model into a more tractable linear one. The linearization makes it much easier to investigate the system's response and design effective controllers, as the resulting system behaves like a set of decoupled double integrators [13]. In classical robotics, inverse dynamics linearization is achieved by using the full dynamic model of the system to cancel out the nonlinear terms and get a linear relationship between input and output.

The equation (2) is linear in control τ, and has full-rank $M(q)$, which can be inverted for any valid configuration. For simplicity in the notation, we set:

$$N(q, \dot{q}) = C(q, \dot{q})\dot{q} + G(q) - \tau_D, \tag{7}$$

taking the control τ as a function of the manipulator state in the form:

$$\tau = M(q)y + N(q, \dot{q}), \tag{8}$$

leads to the system described by:

$$\ddot{q} = y, \tag{9}$$

where y represents a virtual input vector:

$$y = K_D(\dot{q}_{des} - \dot{q}) + K_P(q_{des} - q) + r, \tag{10}$$

where r is the reference component [15].

However, this method requires a real-time knowledge of the changing inertia matrix, Coriolis, and gravitational terms, is computationally expensive and highly state-dependent, especially for aerial manipulators with fast-varying configurations. To avoid computing the true inertia matrix terms online, we propose to use an offline computed fixed nominal model ($\hat{M}, \hat{N}$) in the inverse dynamics controller:

$$\tau = \hat{M}(q)y + \hat{N}(q, \dot{q}). \tag{11}$$

Fig. 1 shows the block diagram of the UAV-manipulator system with application of the approximate inverse dynamics control. For stabilization purposes a Proportional-Derivative (PD) controller is used. By applying control law (10) to our system (8) we get:

$$\ddot{q} = M^{-1}(\widehat{M}y + \widehat{N} - N). \tag{12}$$

The resulting expression can be written as follows:

$$\ddot{q} = y + \left(M^{-1}\widehat{M} - I\right)y + M^{-1}\widetilde{N}, \tag{13}$$

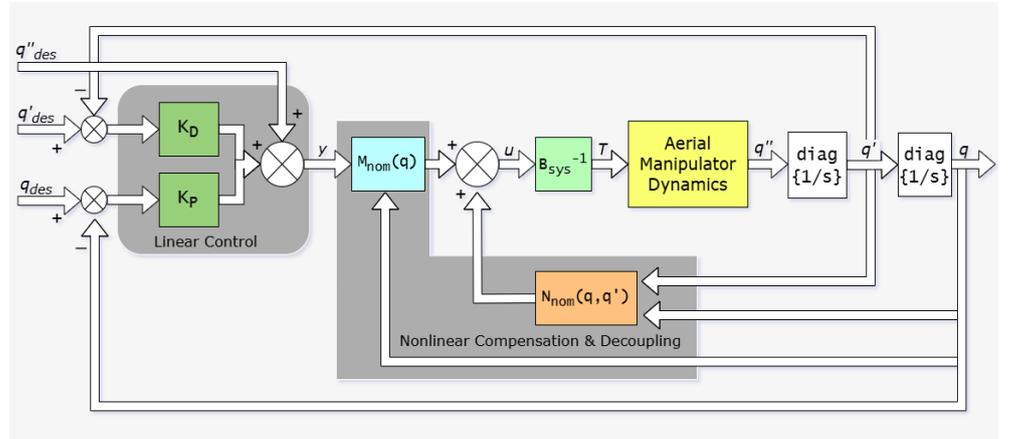$$\widetilde{N}(q, \dot{q}) = \widehat{N}(q, \dot{q}) - N(q, \dot{q}). \tag{14}$$

Fig. 1. The block diagram of the UAV-manipulator system

Thus, we simplified and separated the nonlinear equation into the linear plant and structured uncertainty term δ which shows the deviation between the nominal model and the actual dynamics:

$$\ddot{q} = y + \delta, \tag{15}$$

where δ combines both additive and multiplicative uncertainties:

$$\triangle_a = M^{-1}\widetilde{N}, \tag{16}$$

$$\triangle_m = M^{-1}\widehat{M} - I. \tag{17}$$

Fig. 2 shows the resulting interconnections of double integrator plant and the structured multiplicative and additive uncertainties.
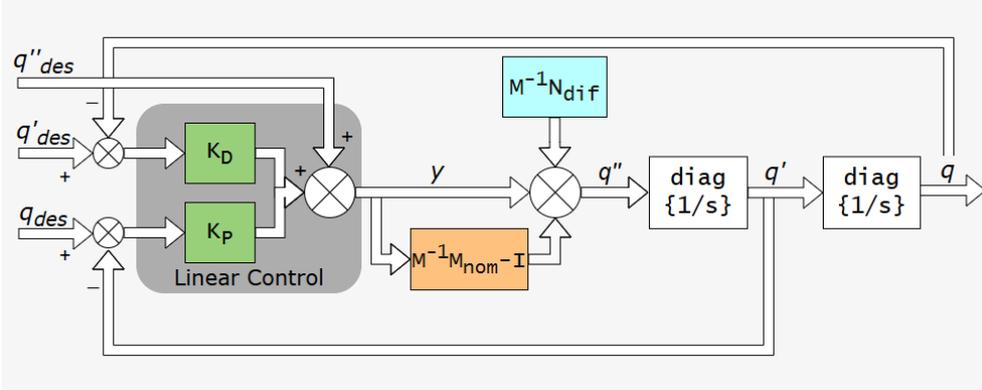
Fig. 2. The resulting block diagram after inverse dynamics linearization

Simulation results. For the validation of the proposed approximate inverse dynamics linearization method, numerical simulations were implemented using the MATLAB software. The main objective of the experiment is to ensure in the effectiveness of the approximate linearization approach, and evaluate the influence of the structured uncertainty terms, and demonstrate the performance of the robust PD controllers under varying conditions of the system.

In the first simulation we investigated the accuracy of the approximate inverse dynmics model. For that purpose, the UAV was commanded to follow some reference trajectory, while the manipulator was actuated through a predefined joint path. The approximate model, which uses the offline-computed nominal inertia matrix, was compared against the full nonlinear dynamics with exact inertia updates.

The simulation scenario involved the UAV tracking a pre-defined square trajectory while manipulator joints executed sinosoidal motions. The simulation results depicted in Fig. 3, show that the approximate inverse dynamics provides sufficiently accurate tracking, with only small deviations from the exact model. These results confirm that the approximate approach captures the essential behavior of the system while avoiding the computational burden of online inertia updates.

In the second simulation, the influence of the uncertainty was analyzed. Fig. 4 illustrates the influence of the structured uncertainty term on each of the system's generalized coordinates. The results demonstrate that the manipulator motion introduces measurable variations in the inertia matrix, which are effectively captured within the structured uncertainty formulation, enabling the application of robust control methods with bounding those uncertainties.

The final simulation was conducted for the timing analysis. Other methods require the online calculation of the state-varying matrices $M(q), C(q, \dot{q})$ and $G(q)$.

As these matrices are generated from complex symbolic derivations, their evaluation is computationally intensive. A timing analysis was performed within the simulation loop to quantify this difference.

The average time for a single evaluation of the full dynamic matrices was measured and compared to the approximate linearization dynamics computation, representing approximately 50 times increase in computational speed. This reduction in computational load makes the proposed approximate linearization scheme not only theoretically but also practically viable for real-time control applications for agile tasks.
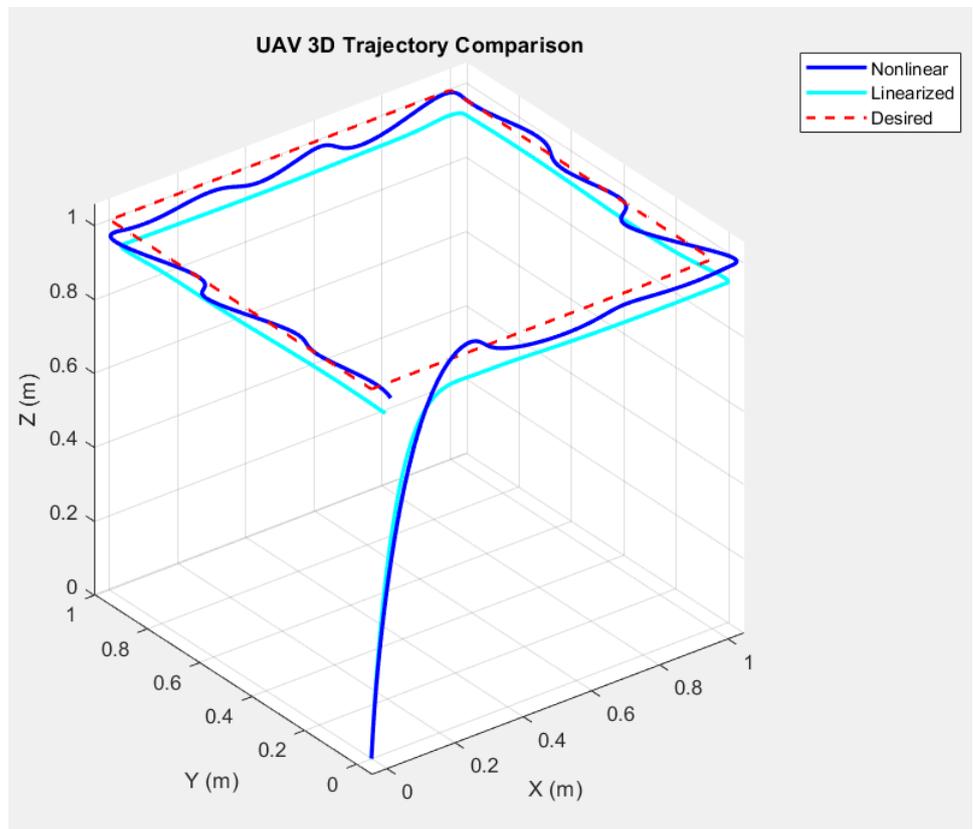

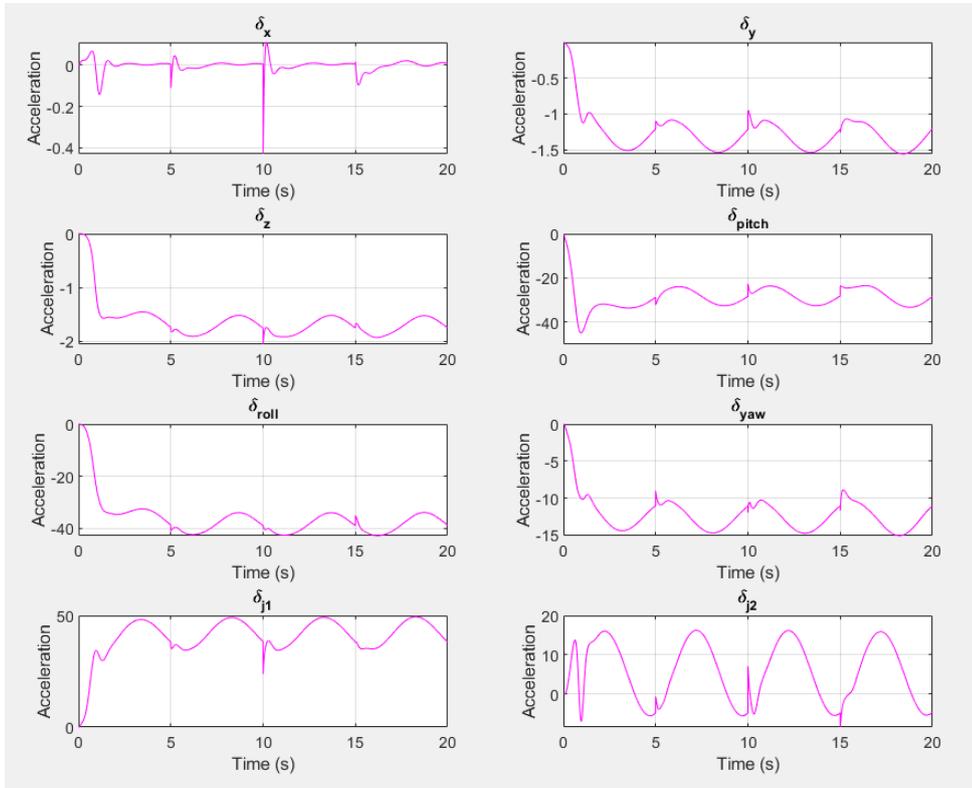*Fig. 3. Trajectory comparison between the exact nonlinear model and the linearized model*

Fig. 4. Influence of the structured uncertainty term on each of the system's generilized co-ordinates

Conclusion. The paper introduces the application of the approximate inverse dynamics linearization to the aerial manipulator system. The simulation results validate the effectiveness of the approximate inverse dynamics linearization approach for aerial manipulator system. We demonstrate that the proposed method not only achieves accurate trajectory tracking but also provides a structure to model and handle the uncertainties of the system. The timing analysis confirms that our approach offers an advantage over methods that require continuous online computation of the full dynamic model.

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ՕԴԱՅԻՆ ՄԱՆԻՊՈՒԼՅԱՏՈՐ ՀԱՄԱԿԱՐԳԻ ՀԱԿԱԴԱՐՁ ԴԻՆԱՄԻԿԱՅԻ ՄՈՏԱՎՈՐ ԳԾԱՅՆԱՑՈՒՄԸ

Տ.Ա. Սիմոնյան

Օդային մանիպուլյատորները մեծ ուշադրություն են գրավում ինչպես օդում մանևրելու, այնպես էլ արտաքին աշխարհի հետ ֆիզիկական փոխազդեցություն ապահովելու շնորհիվ: Այս հատկությունների շնորհիվ՝ դրանք առաջարկում են դժվար հասանելի միջավայրերում խնդիրների արդյունավետ լուծումներ և կիրառվում են տարբեր ոլորտներում՝ ներառյալ ռազմական առաջադրանքները, ենթակառուցվածքների ստուգումը, շինարարությունը, տրանսպորտը և այլն: Այնուամենայնիվ, նման համակարգերի կառավարումը լուրջ խնդիր է՝ պայմանավորված դրանց խիստ ոչ գծային, փոխկապակցված և փոփոխվող դինամիկայով: Կառավարման գոյություն ունեցող մեթոդները, ինչպիսին է հետադարձ կապի գծայնացումը, պահանջում են ամբողջովին հայտնի և անընդհատ թարմացվող դինամիկ մոդել, ինչը մեծ հաշվարկային ծավալ է պահանջում և միշտ չէ, որ իրագործելի է արագ գործնական կիրառությունների համար: Աշխատանքը ներկայացնում է մոտավոր հակադարձ դինամիկայի գծայնացման կիրառումը 2 ազատության աստիճան (ԱԱ) ունեցող ռոբոտ-մանիպուլյատորով հագեցած քառապտուտակ տեսակի անօդաչու թռչող սարքի (ԱԹՍ) դեպքում, որը նախատեսված է տարբեր օգտակար բեռներ բռնելու և տեղափոխելու համար: Մեր մոտեցումն օգտագործում է օֆլայն ռեժիմում հաշվարկված, ֆիքսված նոմինալ արժեքով մոդելի գծայնացումը, դիտարկելով մնացորդային ոչ գծայնությունները և անճշտությունները՝ որպես սահմանափակ կառուցվածքային անորոշություններ: Այս ձևակերպումը պարզեցնում է կառավարման խնդիրը՝ հնարավորություն տալով մշակելու ռոբաստ կարգավորիչներ, որոնք երաշխավորում են կայունություն և բարձր կատարողականություն՝ չնայած համակարգի դինամիկ փոփոխություններին: Թվային մոդելավորումների արդյունքները ցույց տվեցին, որ առաջարկվող մեթոդն ապահովում է հետագծի ճշգրիտ հետևում և արդյունավետորեն հաշվի է առնում մանիպուլյատորի շարժման ազդեցությունը համակարգի դինամիկայի վրա կառուցվածքային անորոշության տարրում: Կարևոր եզրակացություններից է նաև հաշվարկային ծանրաբեռնվածության զգալի կրճատումը. մոտավոր մեթոդը միջինը 50 անգամ ավելի արագ է, քան ամբողջական դինամիկայի առցանց հաշվարկը, ինչն ապացուցում է դրա կիրառելիությունը իրական ժամանակում կառավարման համար:

Առանցքային բառեր. անօդաչու թռչող սարք, ռոբոտ-մանիպուլյատոր, մոտավոր հակադարձ դինամիկա, գծայնացում:

ПРИБЛИЖЕННАЯ ЛИНЕАРИЗАЦИЯ ОБРАТНОЙ ДИНАМИКИ СИСТЕМЫ ЛЕТАЮЩЕГО МАНИПУЛЯТОРА

Т.А. Симонян

Летающие манипуляторы привлекают все большее внимание благодаря своей способности обеспечивать как мобильность в воздухе, так и физическое взаимодей-

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ствие с окружающим миром. Эти качества позволяют использовать их для решения задач в труднодоступных местах, в различных областях, включая военное применение, инспекцию инфраструктуры, строительство и транспорт. Однако управление этими системами связано с серьёзными трудностями из-за их сильно нелинейной, взаимосвязанной и изменяющейся во времени динамики. Существующие методы управления, такие как линеаризация с обратной связью, требуют хорошо известной и постоянно обновляемой динамической модели, которая требует больших вычислительных затрат и не всегда выполнима для быстродействующих реальных приложений. В статье представлено новое применение приближенной линеаризации обратной динамики для беспилотного летательного аппарата (БПЛА)-квадрокоптера, оснащенного роботизированной рукой с 2-мя степенями свободы (СС), предназначенной для захвата и манипулирования различными полезными грузами. Предлагаемый метод использует заранее вычисленную, фиксированную номинальную модель для линеаризации динамики, рассматривая остаточные нелинейности и неточности как ограниченные структурированные неопределенности. Такая формулировка упрощает задачу управления, позволяя создавать робастные контроллеры, обеспечивающие стабильность и производительность, несмотря на динамические изменения в системе. Результаты численного моделирования показывают, что предложенный метод обеспечивает высокую точность слежения за траекторией и эффективно отражает влияние движения манипулятора на динамику системы в структурированной неопределенности. Важным итогом является значительное снижение вычислительной нагрузки: приближенный метод в среднем оказывается примерно в 50 раз быстрее, чем онлайн-расчет полной динамики, что доказывает его пригодность для управления в режиме реального времени.

Ключевые слова: беспилотный летательный аппарат, робот-манипулятор, приближенная обратная динамика, линеаризация.